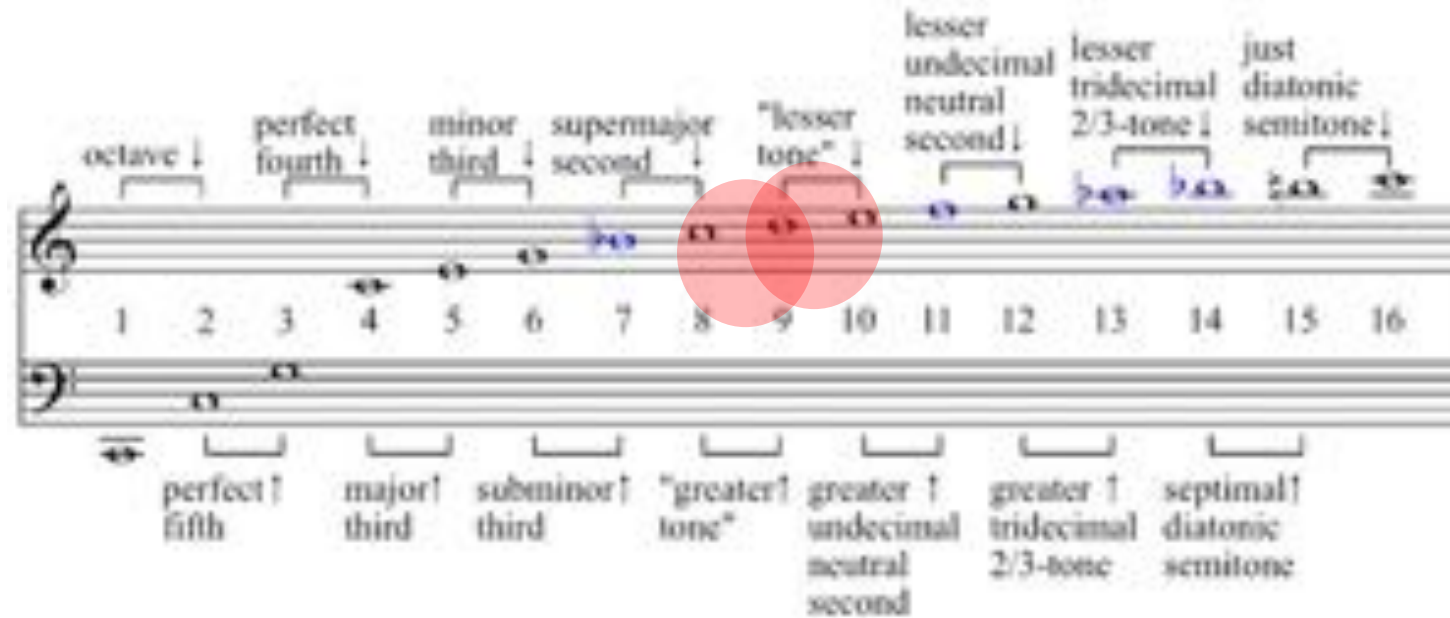


Analysis of musical Intervals based on the early islamic treatises

Amir Abbas Ahmadi

Harmonic Series

Preparation



$$\frac{10}{9} = \frac{5 \times 2}{3 \times 3}$$

$$\frac{9}{8}$$

Cent



204 cent

Cent

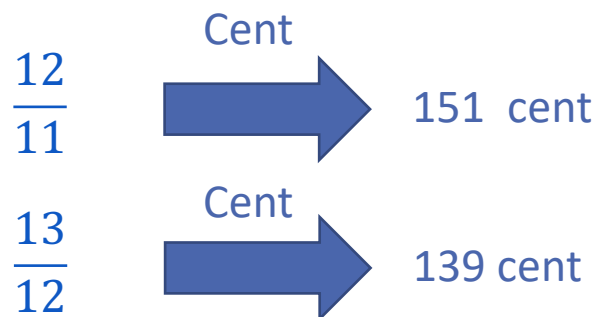
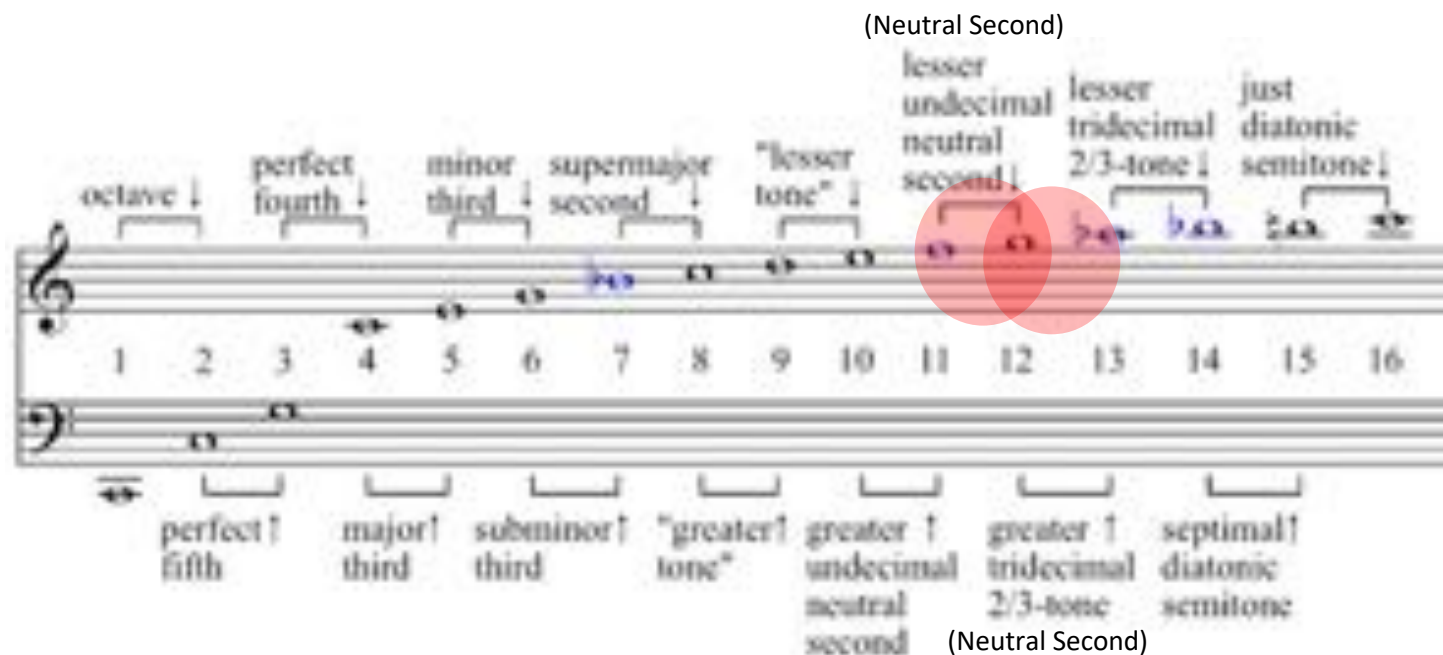


182 cent

$$\frac{10}{9}$$

Preparation

Harmonic Series



Preparation

Notation	Ratio	Cent
A $\flat\flat$	$\frac{2^{18}}{3^{11}} = \frac{262144}{177147}$	678.50
E $\flat\flat$	$\frac{2^{16}}{3^{10}} = \frac{65536}{59049}$	180.45
B $\flat\flat$	$\frac{2^{15}}{3^9} = \frac{32768}{19683}$	882.41
F $\flat$	$\frac{2^{13}}{3^8} = \frac{8192}{6561}$	384.36
C $\flat$	$\frac{2^{11}}{3^7} = \frac{2048}{2187}$	1086.31
G $\flat$	$\frac{2^{10}}{3^6} = \frac{1024}{729}$	588.27
D $\flat$	$\frac{2^8}{3^5} = \frac{256}{243}$	90.23
A $\flat$	$\frac{2^7}{3^4} = \frac{128}{81}$	792.18
E $\flat$	$\frac{2^5}{3^3} = \frac{32}{27}$	294.14
B $\flat$	$\frac{2^4}{3^2} = \frac{16}{9}$	996.09
F	$\frac{2^2}{3^1} = \frac{4}{3}$	498.05
C	$\frac{1}{1}$	0.00

Pythagorean Tuning

$$\text{Fifth} = \frac{3}{2}$$



C $\sharp$	$\frac{3^7}{2^{11}} = \frac{2187}{2048}$	113.69
------------	--	--------

apotome

D $\flat$	$\frac{2^8}{3^5} = \frac{256}{243}$	90.23
-----------	-------------------------------------	-------

Limma

Pythagorean comma

$$113 - 90 = 23 \text{ cent}$$
$$\frac{2187}{2048} \div \frac{256}{243} = \frac{531441}{524288}$$

Notation	Ratio	Cent
C	$\frac{1}{1}$	0.00
G	$\frac{3^1}{2^1} = \frac{3}{2}$	701.96
D	$\frac{3^2}{2^3} = \frac{9}{8}$	203.91
A	$\frac{3^3}{2^4} = \frac{27}{16}$	905.87
E	$\frac{3^4}{2^6} = \frac{81}{64}$	407.82
B	$\frac{3^5}{2^7} = \frac{243}{128}$	1109.78
F $\sharp$	$\frac{3^6}{2^9} = \frac{729}{512}$	611.73
C $\sharp$	$\frac{3^7}{2^{11}} = \frac{2187}{2048}$	113.69
G $\sharp$	$\frac{3^8}{2^{12}} = \frac{6561}{4096}$	815.64
D $\sharp$	$\frac{3^9}{2^{14}} = \frac{19683}{16384}$	317.60
A $\sharp$	$\frac{3^{10}}{2^{15}} = \frac{59049}{32768}$	1019.56
E $\sharp$	$\frac{3^{11}}{2^{17}} = \frac{177147}{131072}$	521.51
B $\sharp$	$\frac{3^{12}}{2^{18}} = \frac{531441}{262144}$	1223.46

Seleucid Empire 312 BC–63 BC and hellenistic period



Persian Empire

Sasanian Empire (224–651 AD)



Umayyaden 661 bis 750 n.ch

- Capital: Damascus



Abbassiden 750 - 1517 n.chr

- Capital: Baghdad



- Tuning systems illustrated in the early texts
- Tuning systems of current music
- comparison

Our historical references

Autor	Treatise
Al-Mausii 767 - 850	???
Al-Kindi 790 - 874	Risala Fi Hubr Ta'lif Al-Alhan
Ibn Hurdadbih ? - 912	Kitab Fi'L-Malahi
Yahya Ibn 'Ali 856 - 912	Risala Fi'L-Musiqi
Sa'adyah Gaon ? - 950	Kitab Al-Amanat
Farabi ? - 950	Kitab Al-Musqi Al-Kabir
Ikhwan Al-Safa Ca. 961 A group of muslim philosophers	scientific treatises
Ali Al-Isfahani 897 - 967	Kitab Al-ghani

Autor	Treatise
Kharizmi ?- 997	Mafatih Al-Ulum
Ibn Sina 980 - 1037	Kitab Al-Shifa Kitab Al-Nijat
Safi Al-Din Urmawi 1216 - 1293	Kitab Al-Adwar Al-Risaöa Al-Sharafiyya
Shirazi ? - 1315	Durrat Al-Tagh
Al-Akfani ? - 1348	Irsad Al-Qasid
Abd Al-Qadir ? - 1435	Maqasid Al-Alhan
Anonym 14. Century	One Treatise

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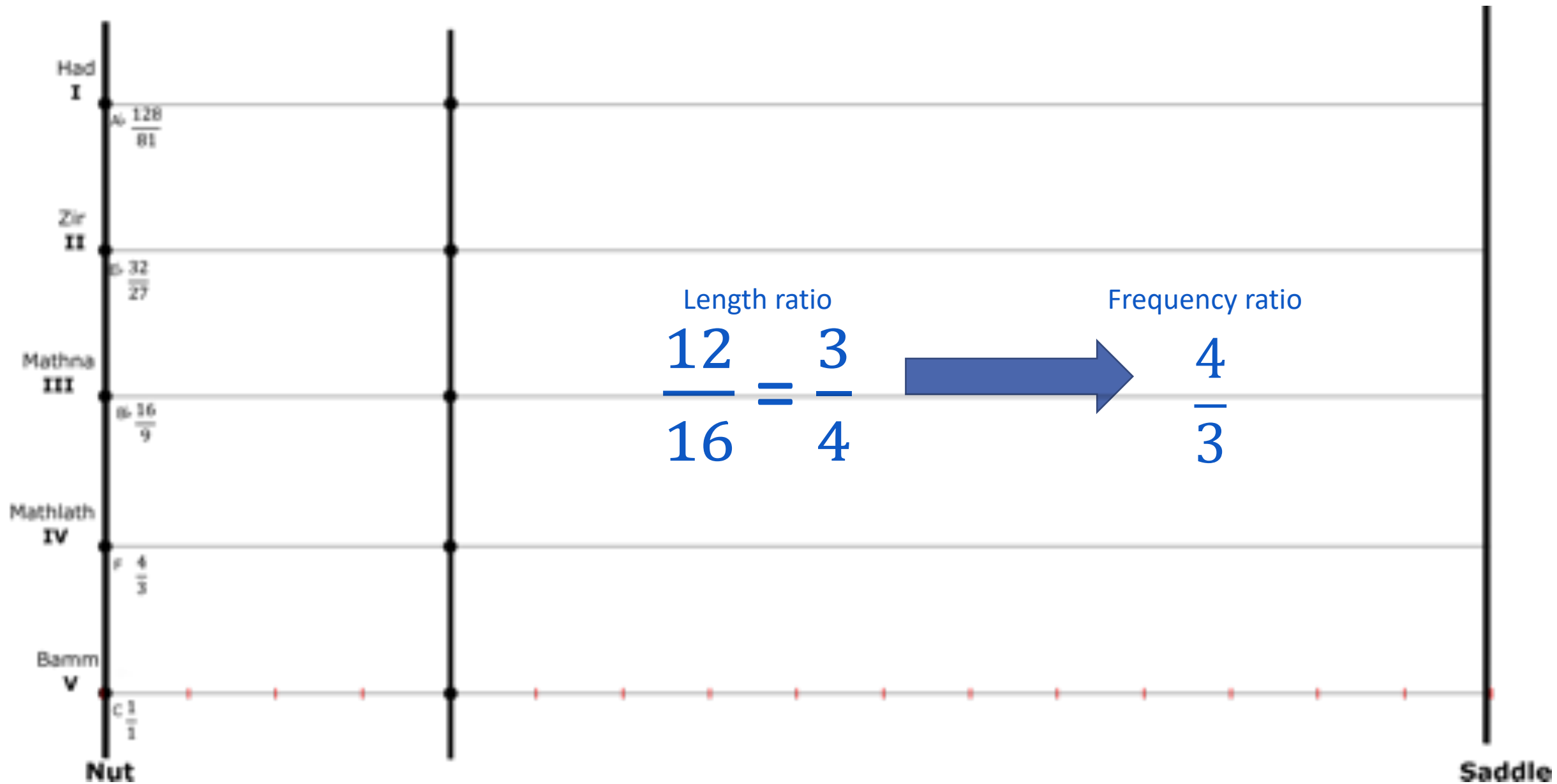


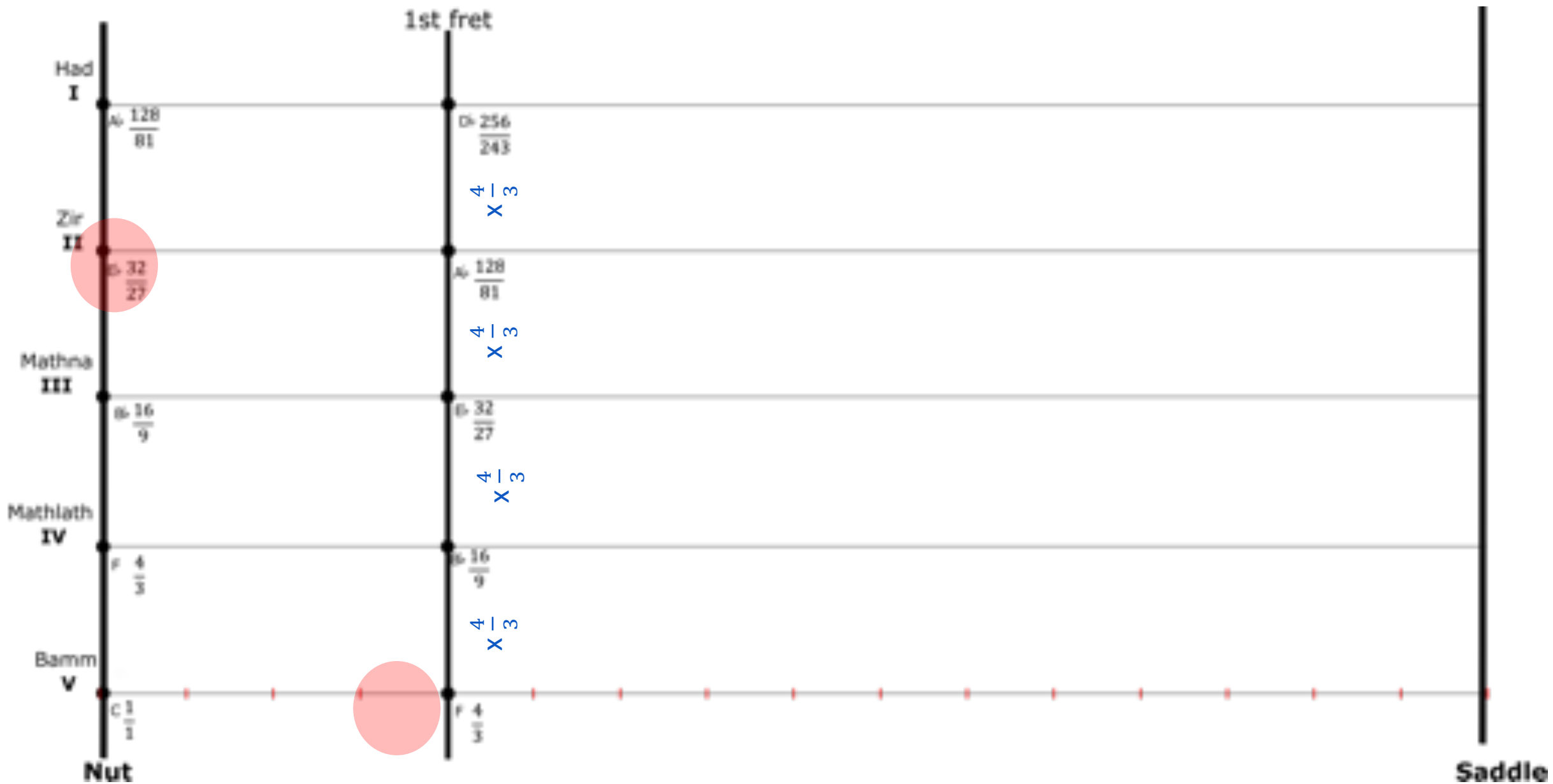
Tuning System of Al-Kindi

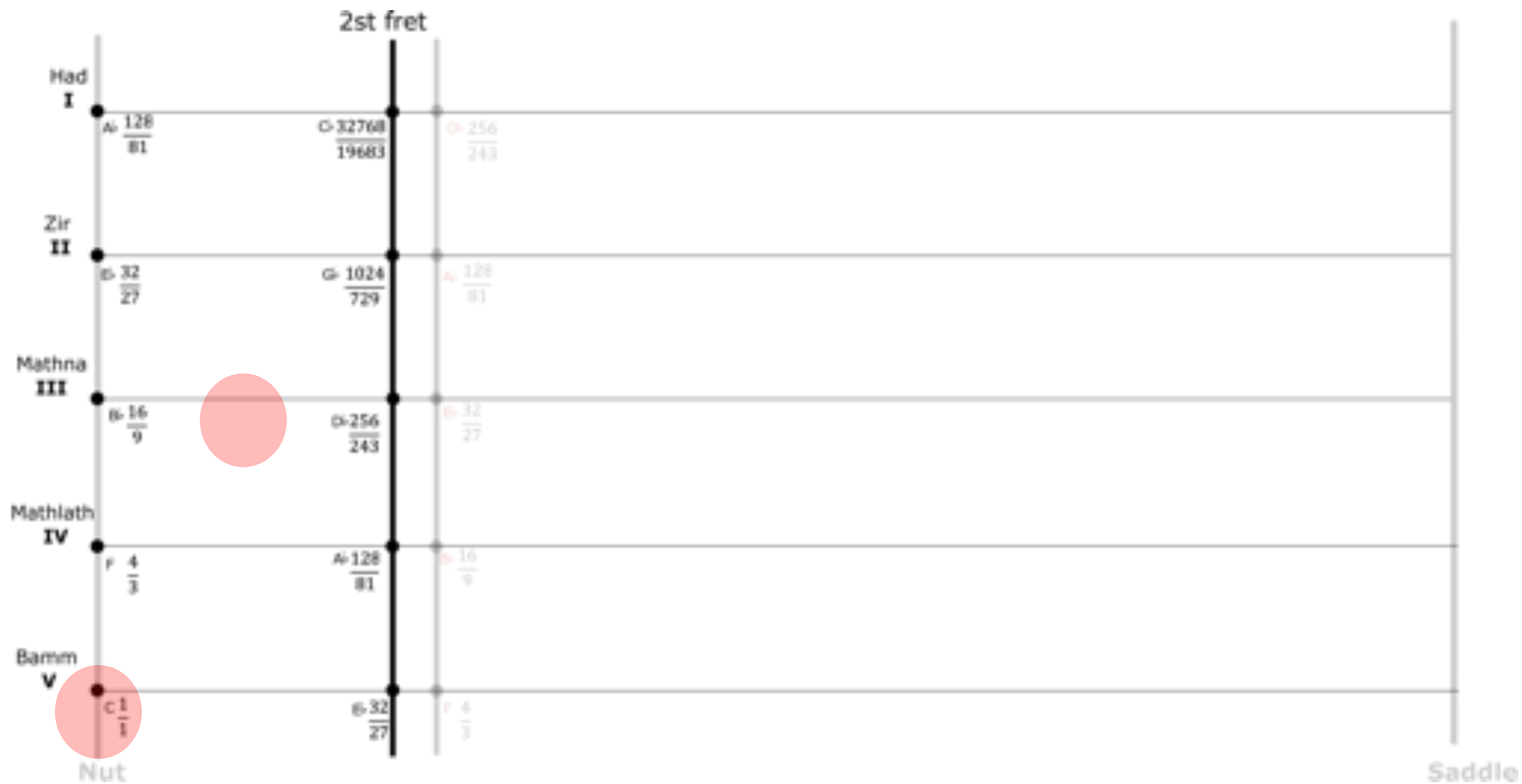
767 - 850

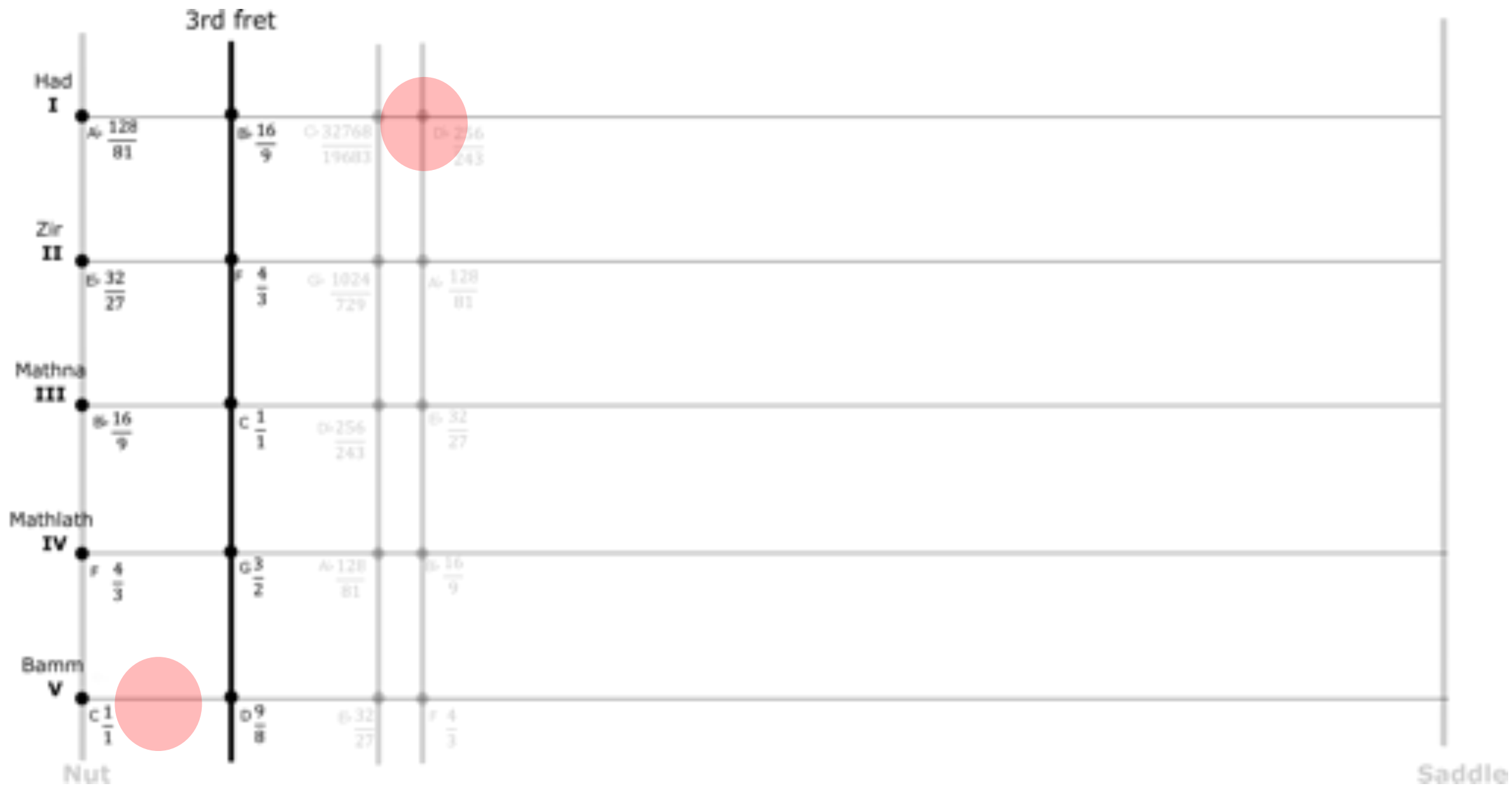
- Risala Fi Hubr Ta'lif Al-Alhan
- The first music treatise of Islam World, which served



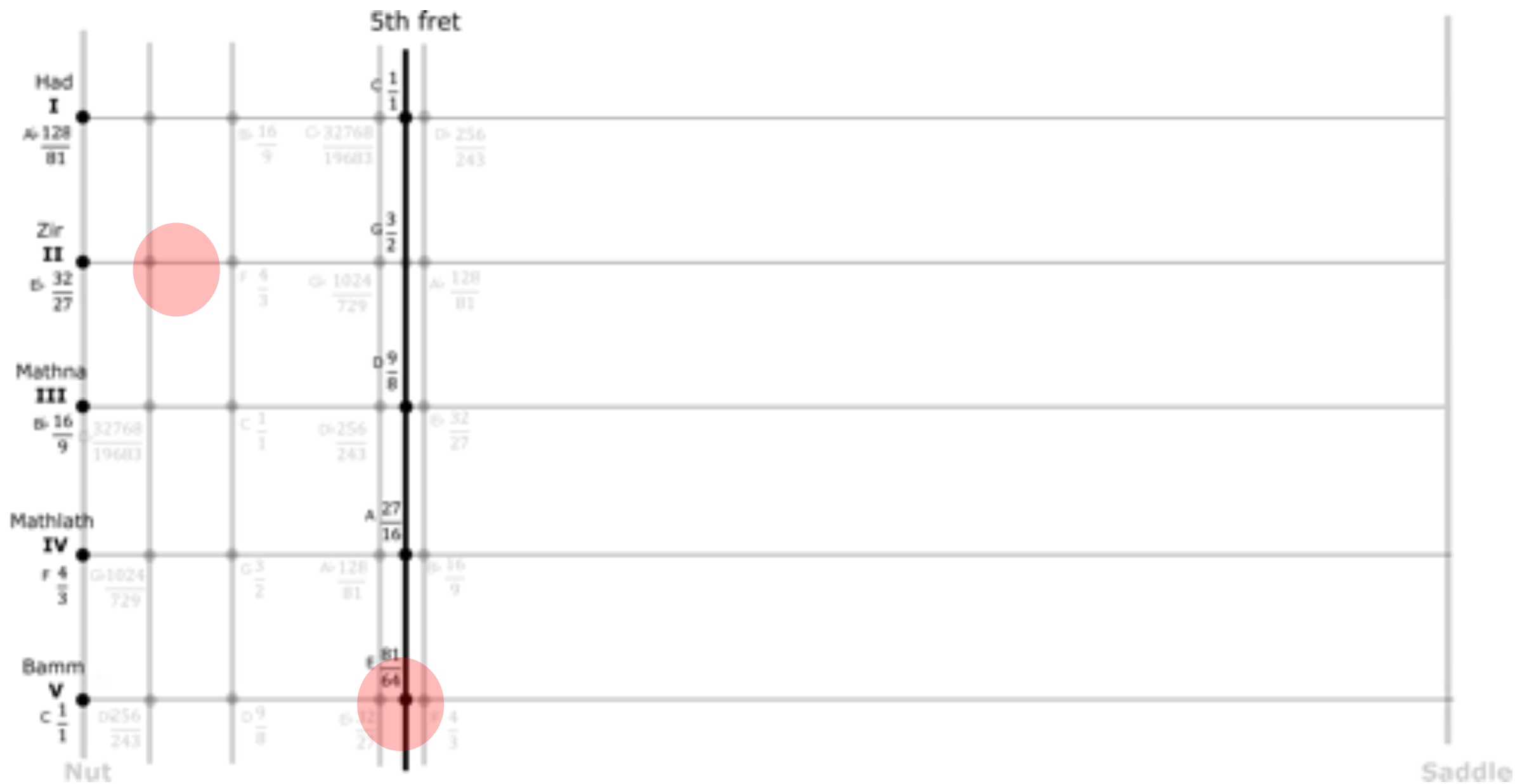




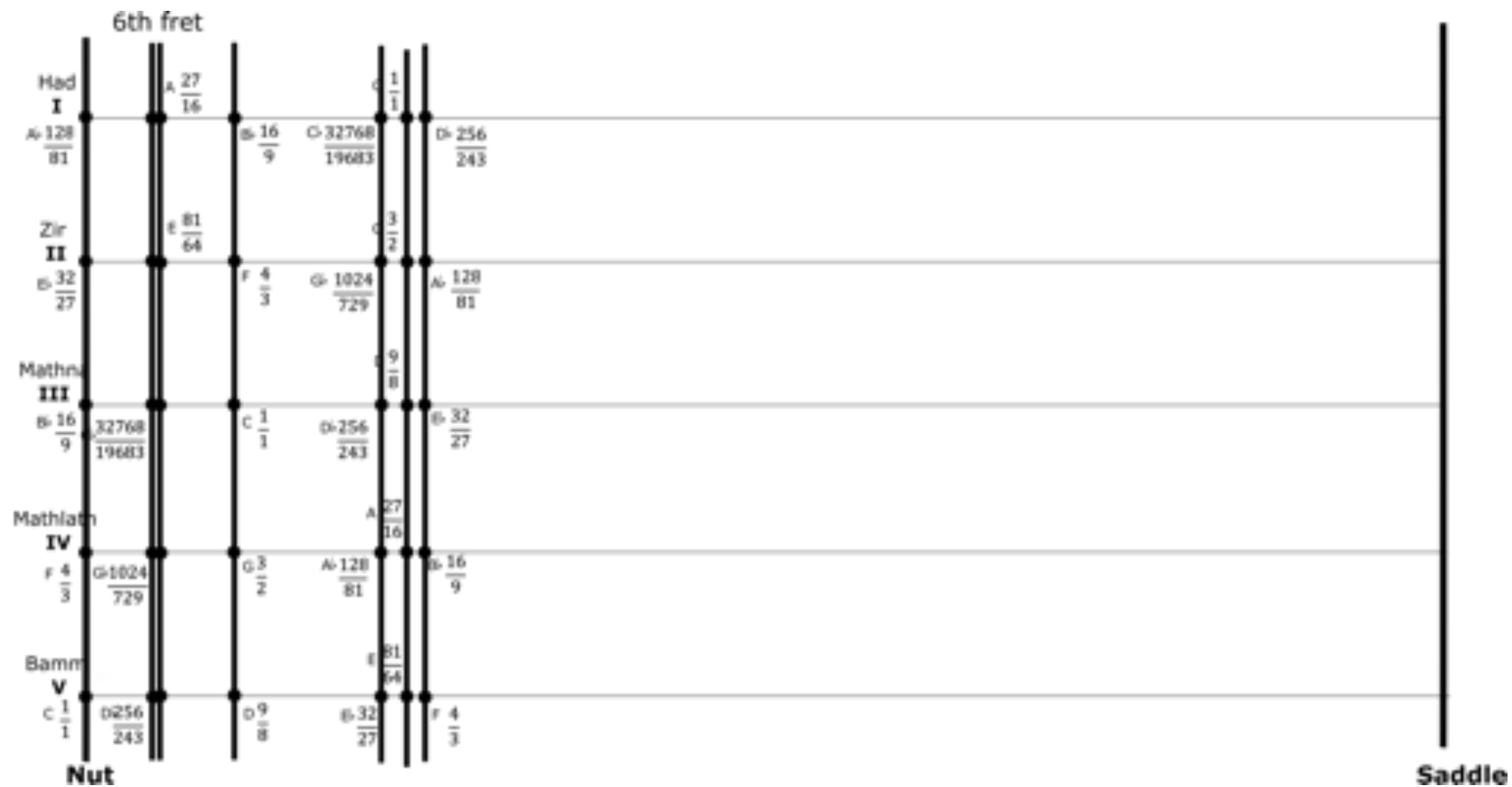


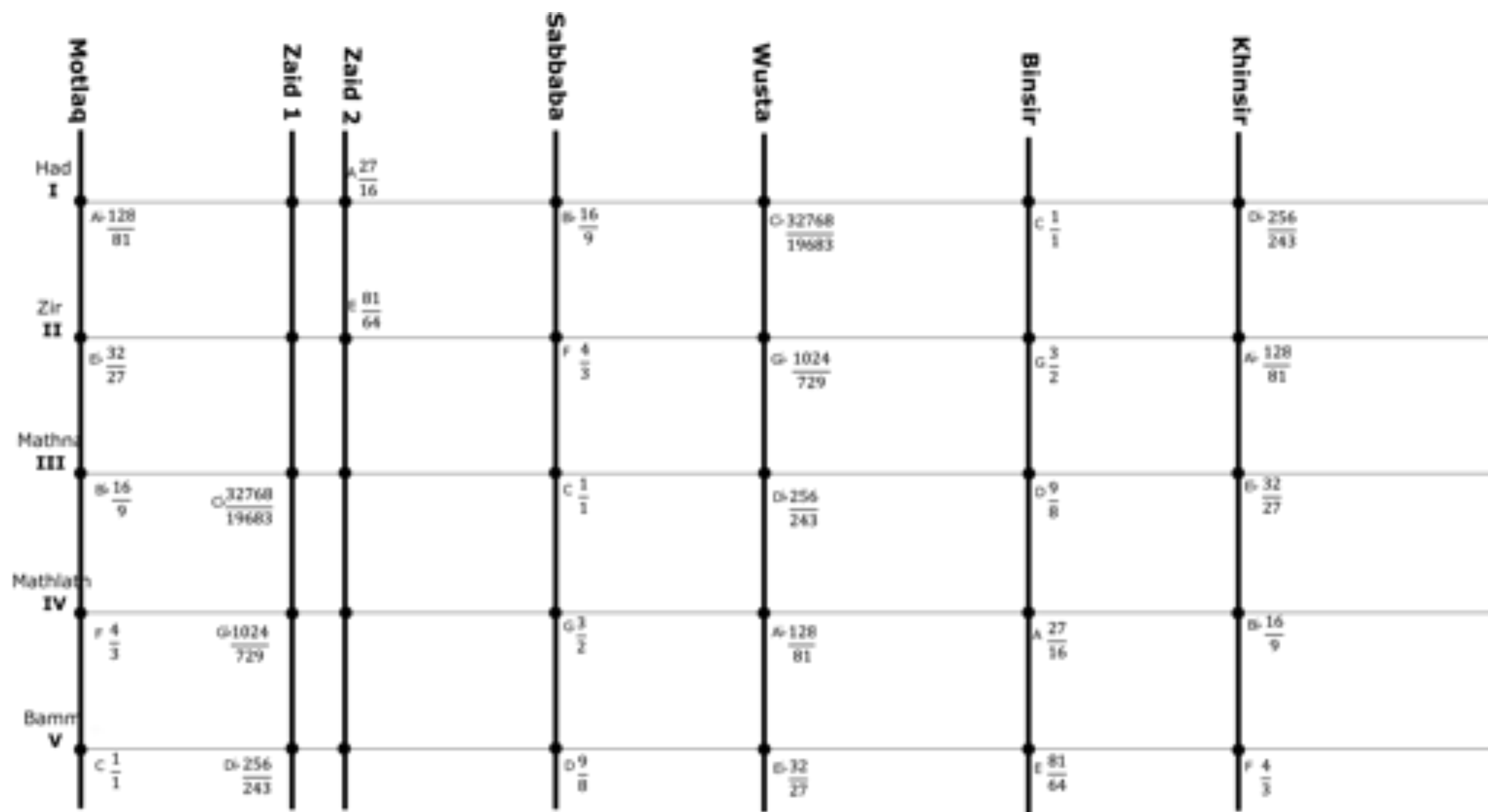


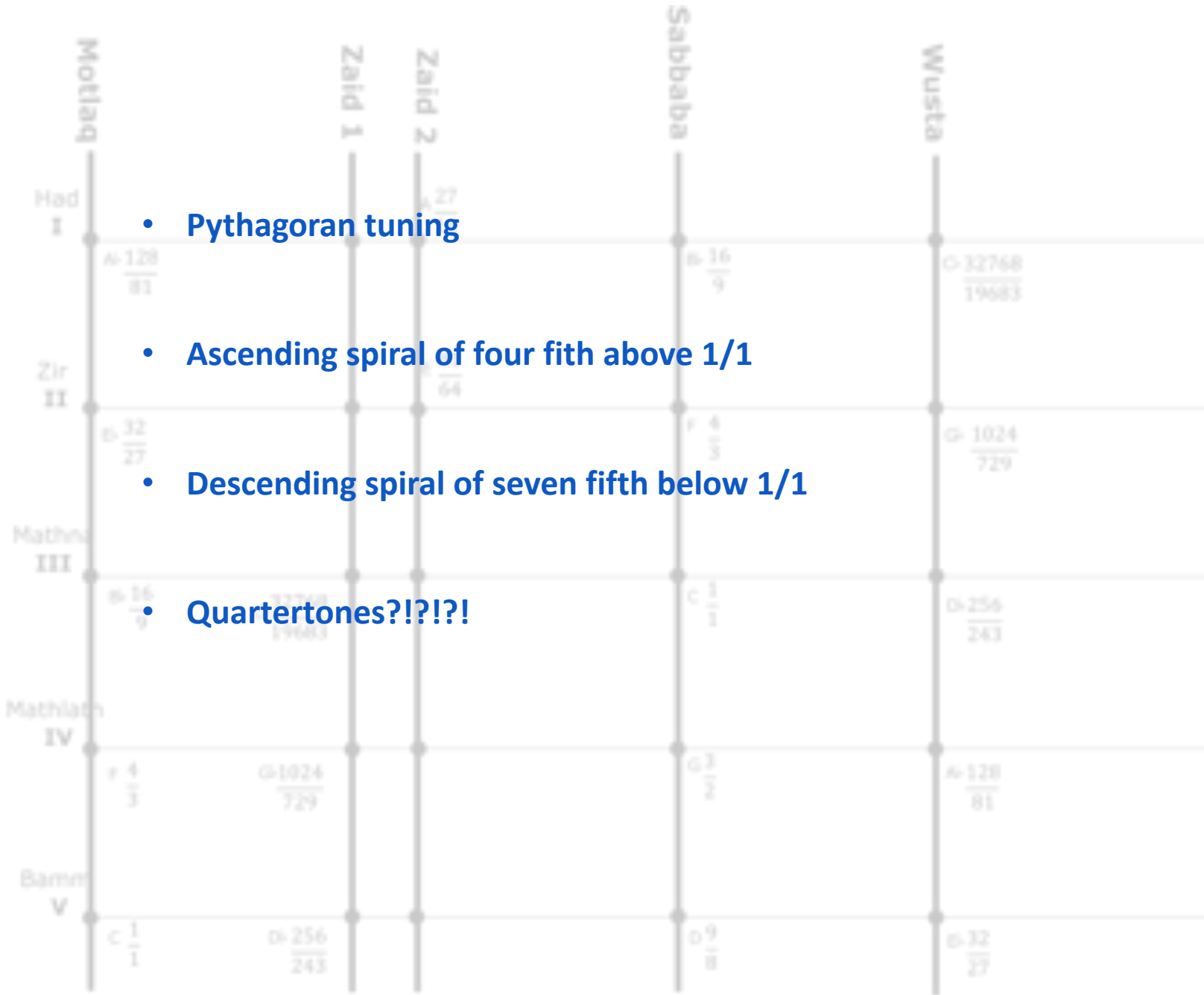












Notation	Ratio	Cent
E	$\frac{3^4}{2^6} = \frac{81}{64}$	408
A	$\frac{3^3}{2^4} = \frac{27}{16}$	906
D	$\frac{3^2}{2^3} = \frac{9}{8}$	204
G	$\frac{3^1}{2^1} = \frac{3}{2}$	702
C	$\frac{1}{1}$	0
F	$\frac{2^2}{3^1} = \frac{4}{3}$	498
B $\flat$	$\frac{2^4}{3^2} = \frac{16}{9}$	996
E $\flat$	$\frac{2^5}{3^3} = \frac{32}{27}$	294
A $\flat$	$\frac{2^7}{3^4} = \frac{128}{81}$	792
D $\flat$	$\frac{2^8}{3^5} = \frac{256}{243}$	90
G $\flat$	$\frac{2^{10}}{3^6} = \frac{1024}{729}$	588
C $\flat$	$\frac{2^{11}}{3^7} = \frac{2048}{2187}$	1086



Tuning System of Farabi (872 ? – 950)

Kitab Al-Musqi Al-Kabir:



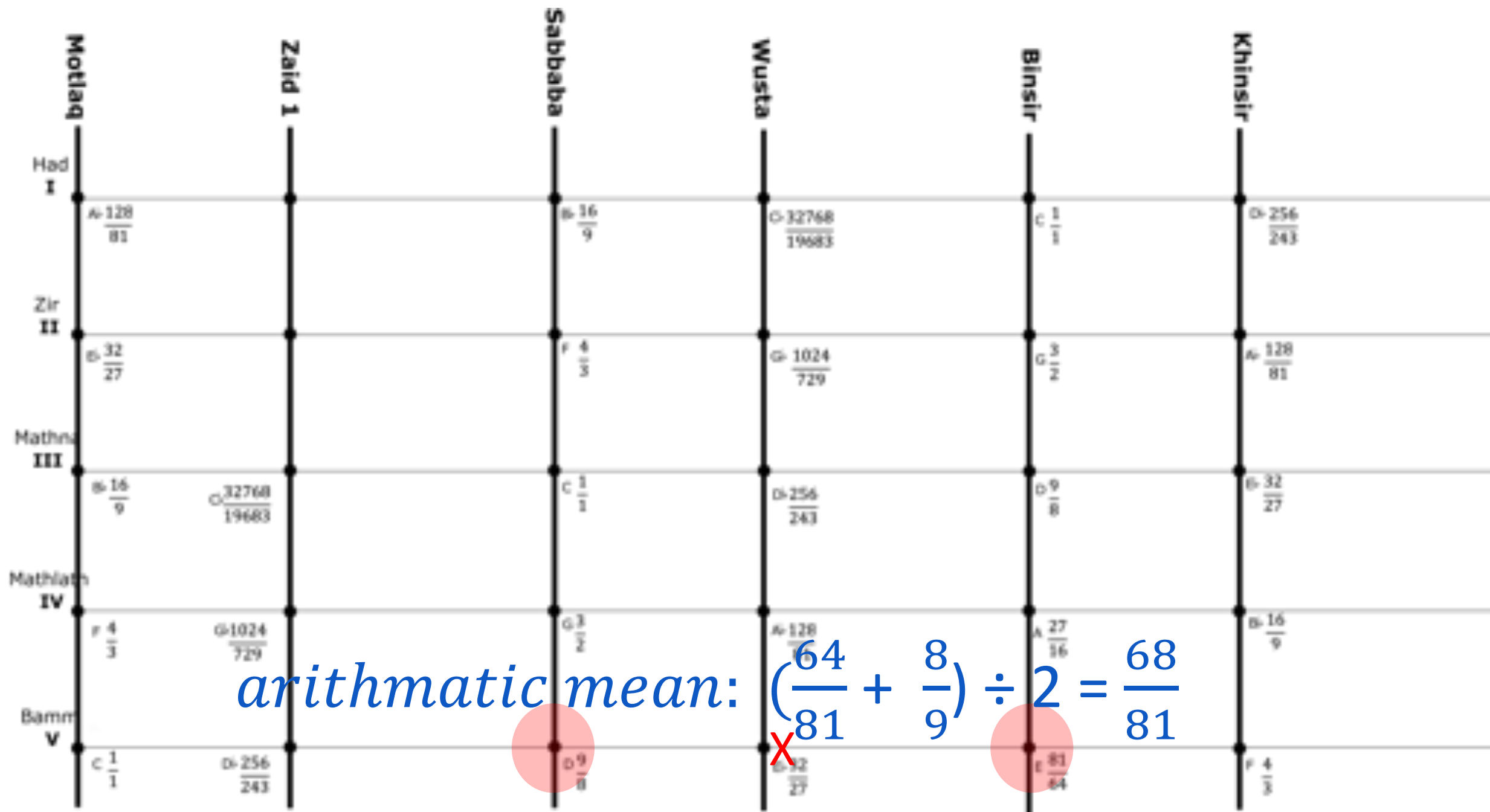
- Tuning System of Oud

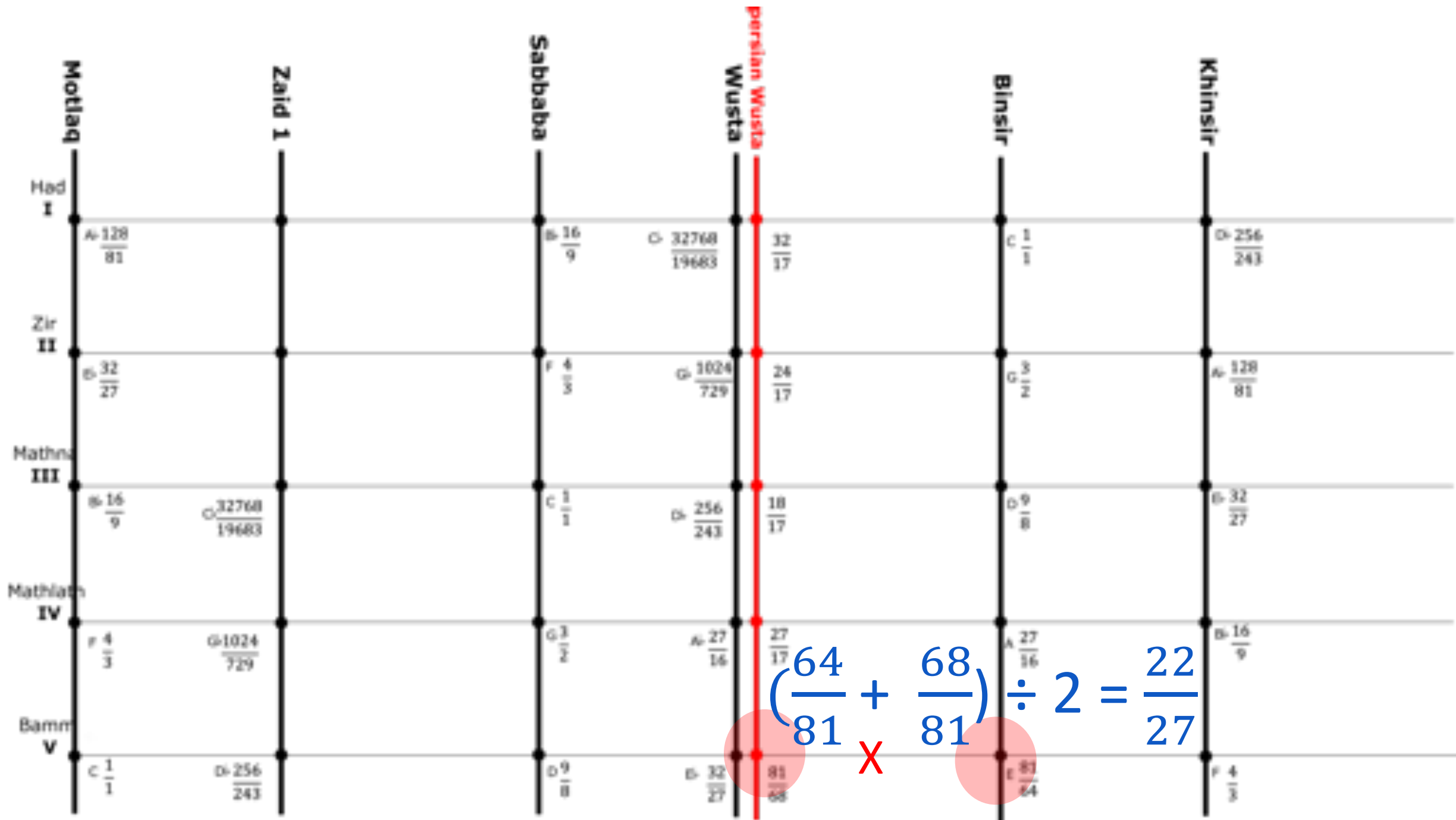


- Tuning system of Tanbur of Khorasan

- Tuning System of Tanbur of Baghdad

- Tuning System of Ney and Rabbab





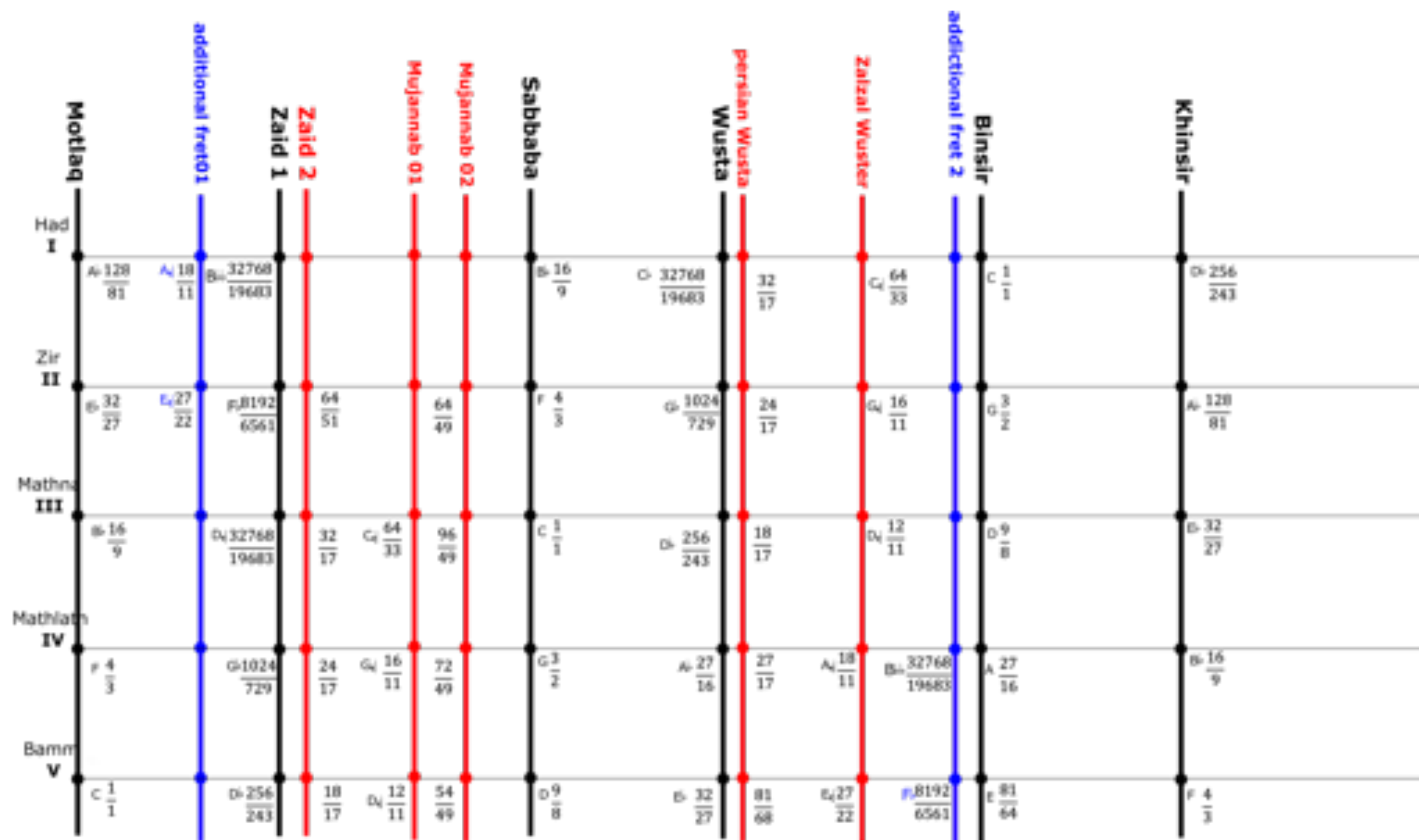


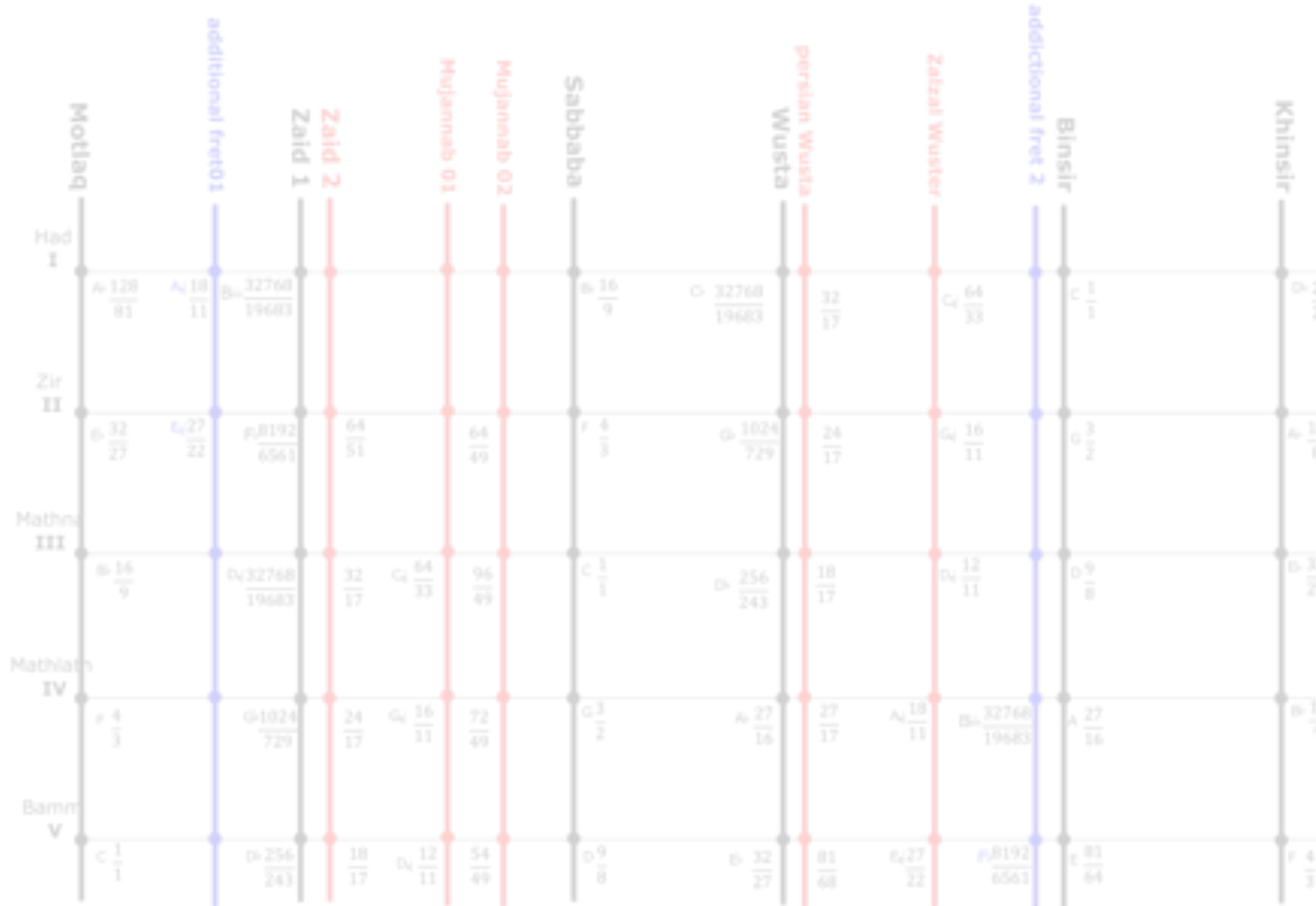
	Motlaq	Zaid 1	Zaid 2	Sabbaba	perjan Wusta Wusta	Zalzal Wuster	Blinsir	Khinsir
Had I	$A = \frac{128}{81}$	$B = \frac{32768}{19683}$		$B = \frac{16}{9}$	$C = \frac{32768}{19683}$	$C = \frac{64}{33}$	$C = \frac{1}{11}$	$D = \frac{256}{243}$
Zir II	$B = \frac{32}{27}$	$F = \frac{8192}{6561}$	$\frac{64}{51}$	$F = \frac{4}{3}$	$G = \frac{1024}{729}$	$G = \frac{16}{11}$	$G = \frac{8}{23}$	$A = \frac{128}{81}$
Mathna III	$B = \frac{16}{9}$	$C = \frac{32768}{19683}$	$\frac{32}{17}$	$C = \frac{1}{1}$	$D = \frac{256}{243}$	$D = \frac{12}{11}$	$D = \frac{9}{8}$	$B = \frac{32}{27}$
Mathlath IV	$F = \frac{4}{3}$	$G = \frac{1024}{729}$	$\frac{24}{17}$	$G = \frac{3}{2}$	$A = \frac{27}{16}$	$A = \frac{18}{11}$	$A = \frac{27}{16}$	$B = \frac{16}{9}$
Bamm V	$C = \frac{1}{1}$	$D = \frac{256}{243}$	$\frac{18}{17}$	$D = \frac{9}{8}$	$B = \frac{32}{27}$	$E = \frac{27}{22}$	$E = \frac{81}{64}$	$F = \frac{4}{3}$



	Motlaq	Zaid 1	Zaid 2	Mujannab 01	Mujannab 02	Sabbaba	perjan Wusta Wusta	Zalzal Wuster	Blinsir	Khinsir
Had I	$A = \frac{128}{81}$	$B = \frac{32768}{19683}$				$B = \frac{16}{9}$	$C = \frac{32768}{19683}$ $\frac{32}{17}$	$C = \frac{64}{33}$	$C = \frac{1}{11}$	$D = \frac{256}{243}$
Zir II	$B = \frac{32}{27}$	$F = \frac{8192}{6561}$	$\frac{64}{51}$	$F = \frac{192}{149}$ $\frac{64}{49}$		$F = \frac{4}{3}$	$G = \frac{1024}{729}$ $\frac{24}{17}$	$G = \frac{16}{11}$	$G = \frac{8}{24}$	$A = \frac{128}{81}$
Mathna III	$B = \frac{16}{9}$	$G = \frac{32768}{19683}$	$\frac{32}{17}$	$C = \frac{288}{149}$ $\frac{96}{49}$		$C = \frac{1}{1}$	$D = \frac{256}{243}$ $\frac{18}{17}$	$D = \frac{12}{11}$	$D = \frac{9}{8}$	$B = \frac{32}{27}$
Mathlath IV	$F = \frac{4}{3}$	$G = \frac{1024}{729}$	$\frac{24}{17}$	$G = \frac{216}{149}$ $\frac{72}{49}$		$G = \frac{3}{2}$	$A = \frac{27}{16}$ $\frac{27}{17}$	$A = \frac{18}{11}$	$A = \frac{27}{16}$	$B = \frac{16}{9}$
Bamm V	$C = \frac{1}{1}$	$D = \frac{256}{243}$	$\frac{18}{17}$	$D = \frac{162}{149}$ $\frac{54}{49}$		$D = \frac{9}{8}$	$B = \frac{32}{27}$ $\frac{81}{68}$	$E = \frac{27}{22}$	$E = \frac{81}{64}$	$F = \frac{4}{3}$







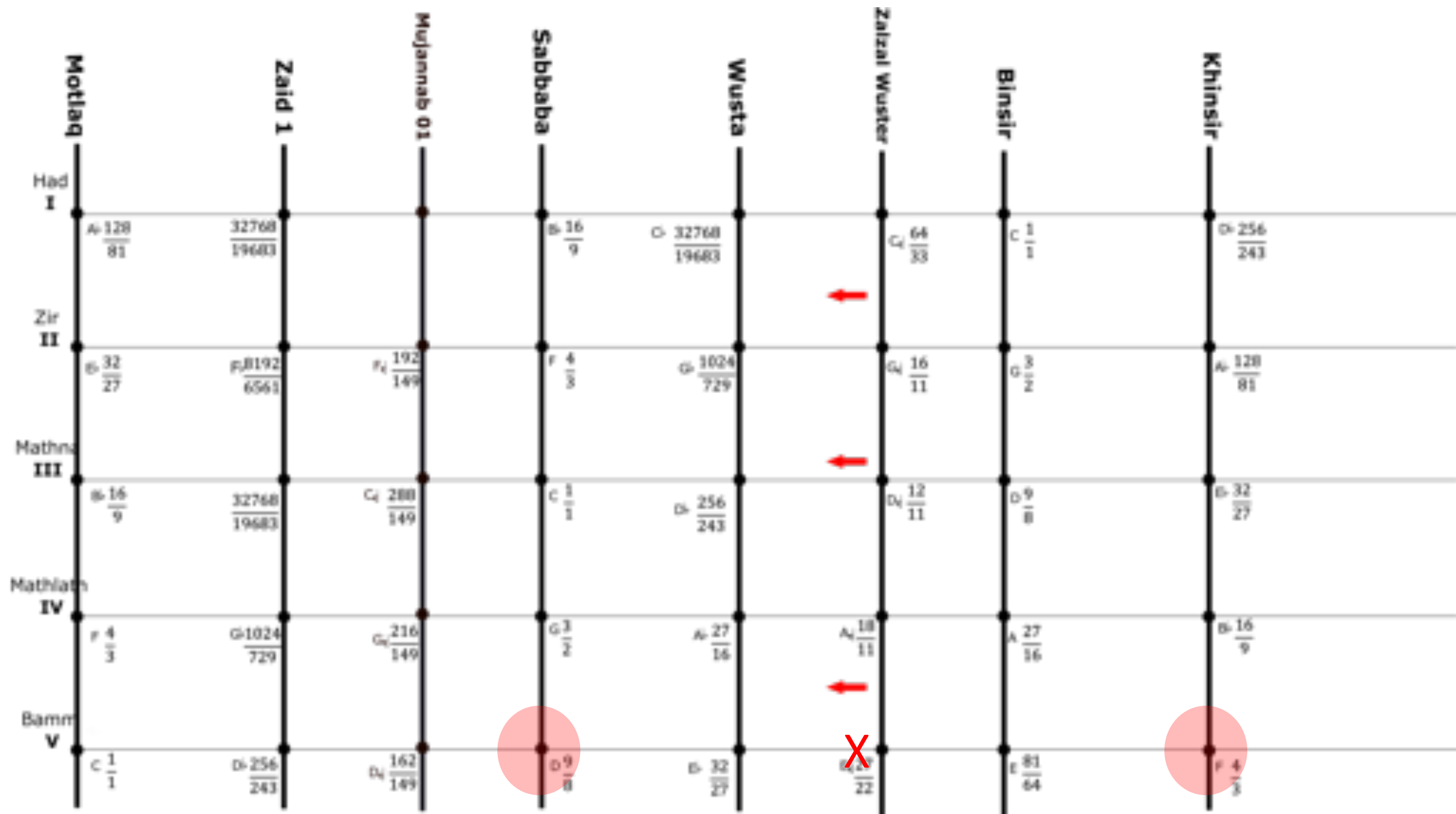
Notation	Ratio	Cent
C	1/1	0
D $\flat$	256/243	90
D $\flat$	18/17	99
D $\natural$	12/11	151
D	9/8	204
E $\flat$	32/27	294
E $\natural$	27/22	355
F $\flat$	221/177	384
E	81/64	408
F	4/3	498
G $\flat$	1024/729	588
G $\flat$	24/17	597
G	3/2	702
A $\flat$	128/81	792
A $\natural$	18/11	853
B $\flat\flat$	32768/19683	882
A	27/16	906
B $\flat$	16/9	996
C $\flat$	221/118	1086
C $\flat$	32/17	1095
C $\natural$	64/33	1147
C	2/1	1200



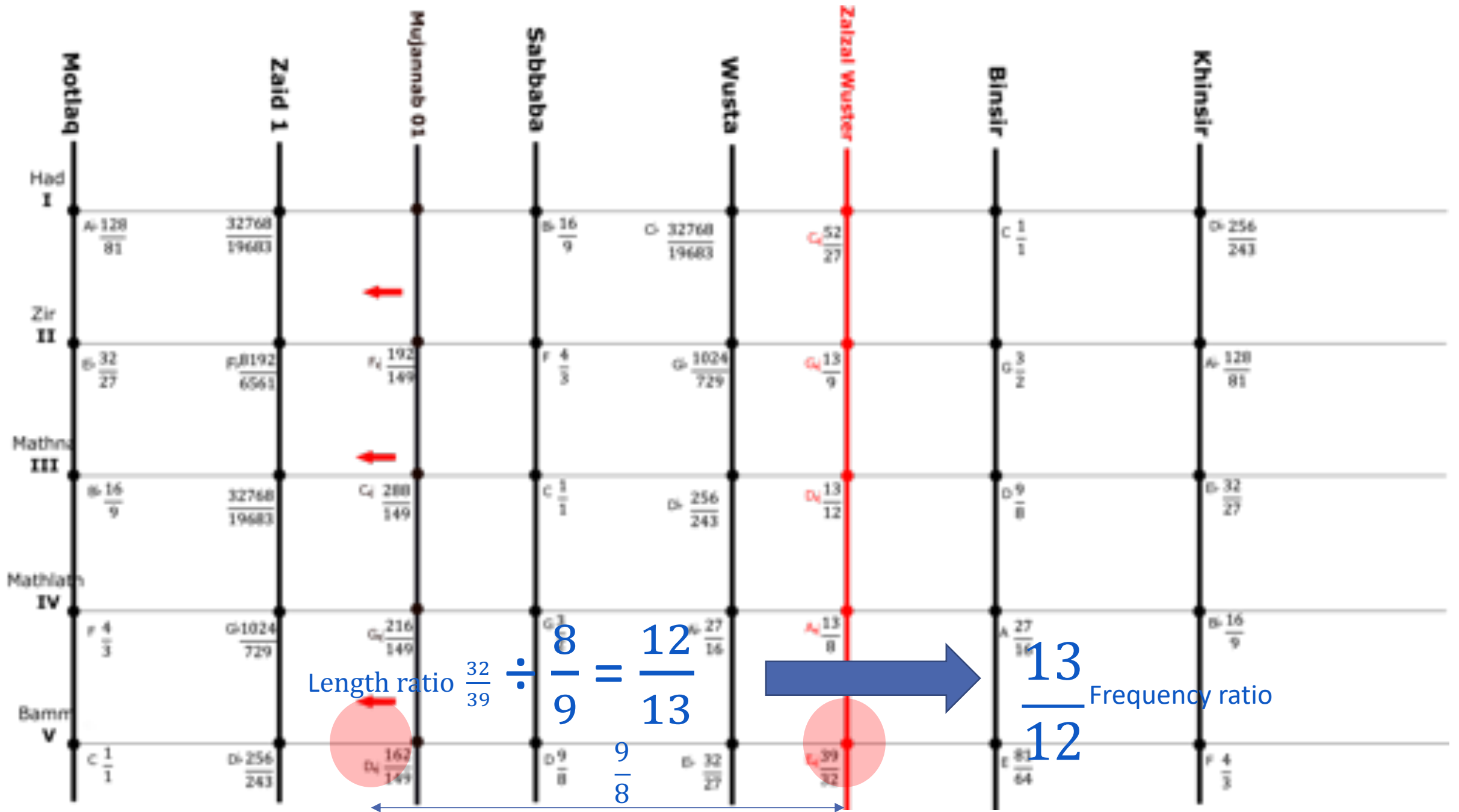
Tuning System of Ibn Sina (Avicenna) (980–1037)

- Kitab Al-Shifa
- Kitab Al-Nijat

	Motlaq	Zaid 1	Mujannab 01	Sabbaba	Wusta	Zalzai Wuster	Binsir	Khinsir
Had I	$A \frac{128}{81}$	$\frac{32768}{19683}$		$B \frac{16}{9}$	$G \frac{32768}{19683}$	$C \frac{64}{33}$	$C \frac{1}{1}$	$D \frac{256}{243}$
Zir II	$B \frac{32}{27}$	$F \frac{8192}{6561}$	$F \frac{192}{149}$	$F \frac{4}{3}$	$G \frac{1024}{729}$	$G \frac{16}{11}$	$G \frac{3}{2}$	$A \frac{128}{81}$
Mathna III	$B \frac{16}{9}$	$\frac{32768}{19683}$	$C \frac{288}{149}$	$C \frac{1}{2}$	$D \frac{256}{243}$	$D \frac{12}{11}$	$D \frac{9}{8}$	$B \frac{32}{27}$
Mathlath IV	$F \frac{4}{3}$	$G \frac{1024}{729}$	$G \frac{216}{149}$	$G \frac{3}{2}$	$A \frac{27}{16}$	$A \frac{18}{11}$	$A \frac{27}{16}$	$B \frac{16}{9}$
Bamm V	$C \frac{1}{1}$	$D \frac{256}{243}$	$D \frac{162}{149}$	$D \frac{9}{8}$	$B \frac{32}{27}$	$E \frac{27}{22}$	$E \frac{81}{64}$	$F \frac{4}{3}$



	Motlaq	Zaid 1	Mujannab 01	Sabbaba	Wusta	Zalzai Wuster	Binsir	Khinsir
Had I	$A \frac{128}{81}$	$\frac{32768}{19683}$		$B \frac{16}{9}$	$G \frac{32768}{19683}$	$C \frac{52}{27}$	$C \frac{1}{1}$	$D \frac{256}{243}$
Zir II	$B \frac{32}{27}$	$F \frac{8192}{6561}$	$F_4 \frac{192}{149}$	$F \frac{4}{3}$	$G \frac{1024}{729}$	$G \frac{13}{9}$	$G \frac{8}{27}$	$A \frac{128}{81}$
Mathna III	$B \frac{16}{9}$	$\frac{32768}{19683}$	$C_4 \frac{288}{149}$	$C \frac{1}{1}$	$D \frac{256}{243}$	$D \frac{13}{12}$	$D \frac{9}{8}$	$B \frac{32}{27}$
Mathlath IV	$F \frac{4}{3}$	$G \frac{1024}{729}$	$G_4 \frac{216}{149}$	$G \frac{3}{2}$	$A \frac{27}{16}$	$A \frac{13}{8}$	$A \frac{27}{16}$	$B \frac{16}{9}$
Bamm V	$C \frac{1}{1}$	$D \frac{256}{243}$	$D_4 \frac{162}{149}$	$D \frac{9}{8}$	$B \frac{32}{27}$	$E \frac{39}{32}$	$E \frac{81}{64}$	$F \frac{4}{3}$

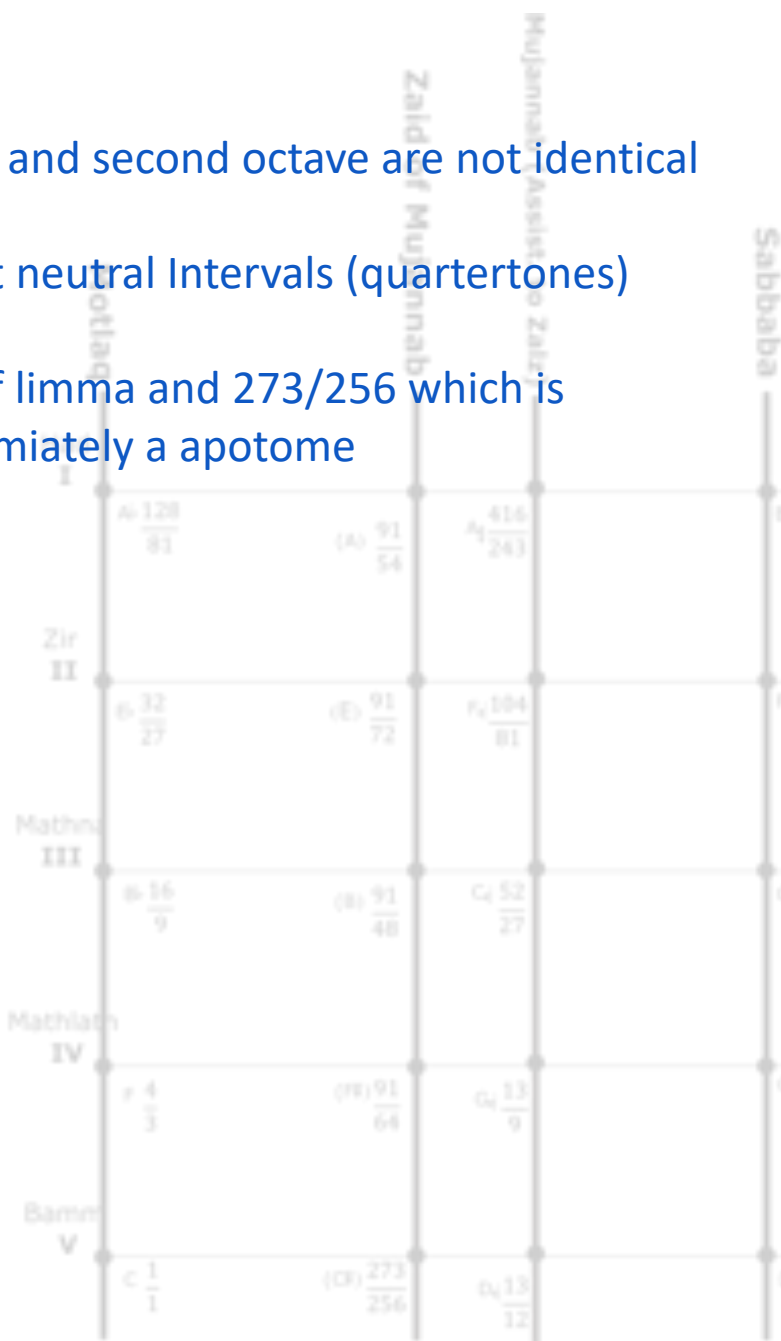


	Motlaq	Zaid 1	Mujannab (Assist. to Zal)	Sabbaba	Wusta	Zalzal Wuster	Binsir	Khinsir
Had I	$A_1 \frac{128}{81}$	$\frac{32768}{19683}$	$A_2 \frac{416}{243}$	$B_1 \frac{16}{9}$	$D_1 \frac{32768}{19683}$	$C_2 \frac{52}{27}$	$C_1 \frac{1}{1}$	$D_1 \frac{256}{243}$
Zir II	$B_1 \frac{32}{27}$	$F_1 \frac{8192}{6561}$	$F_2 \frac{104}{81}$	$F_1 \frac{4}{3}$	$D_1 \frac{1024}{729}$	$G_2 \frac{13}{9}$	$G_1 \frac{3}{2}$	$A_1 \frac{128}{81}$
Mathna III	$B_1 \frac{16}{9}$	$\frac{32768}{19683}$	$C_2 \frac{52}{27}$	$C_1 \frac{1}{1}$	$D_1 \frac{256}{243}$	$D_2 \frac{13}{12}$	$D_1 \frac{9}{8}$	$B_1 \frac{32}{27}$
Mathlath IV	$F_1 \frac{4}{3}$	$G_1 \frac{1024}{729}$	$G_2 \frac{13}{9}$	$G_1 \frac{3}{2}$	$A_1 \frac{27}{16}$	$A_2 \frac{13}{8}$	$A_1 \frac{27}{16}$	$B_1 \frac{16}{9}$
Bamm V	$C_1 \frac{1}{1}$	$D_1 \frac{256}{243}$	$D_2 \frac{13}{12}$	$D_1 \frac{9}{8}$	$B_1 \frac{32}{27}$	$E_2 \frac{39}{32}$	$E_1 \frac{81}{64}$	$F_1 \frac{4}{3}$



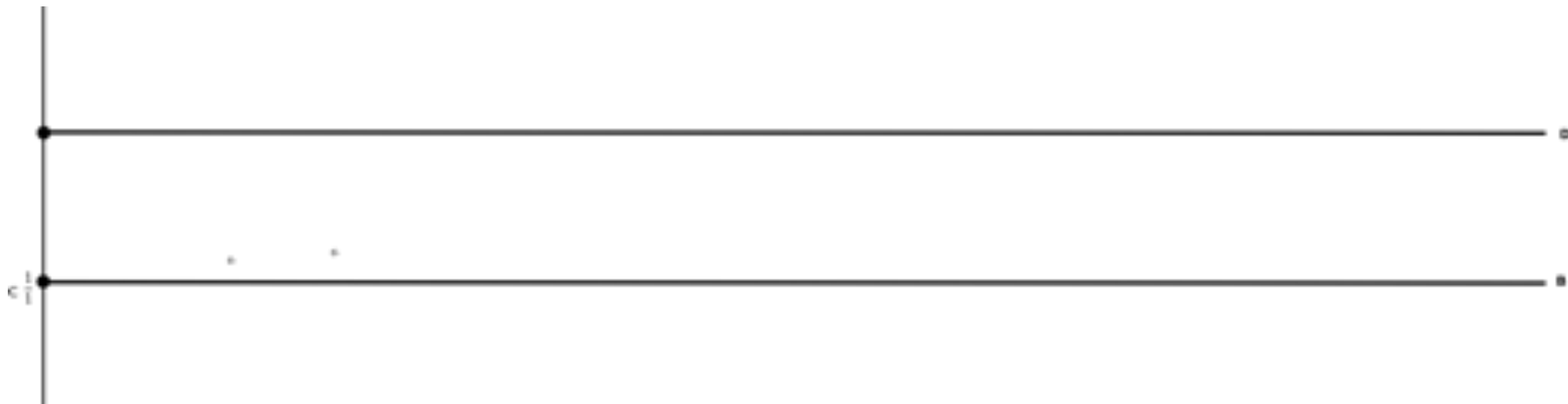
	Motlaq	Zaid of Mujannab	Mujannab (Assist. to Zalx)	Sabbaba	Wusta	Zalzal Wuster	Binsir	Khinsir
Had I	$A_1 \frac{128}{81}$	$(A_1) \frac{91}{54}$	$A_1 \frac{416}{243}$	$B_1 \frac{16}{9}$	$O \frac{32768}{19683}$	$C_1 \frac{52}{27}$	$C \frac{1}{1}$	$O \frac{256}{243}$
Zir II	$B_1 \frac{32}{27}$	$(B_1) \frac{91}{72}$	$F_1 \frac{104}{81}$	$F \frac{4}{3}$	$O \frac{1024}{729}$	$G_1 \frac{13}{9}$	$G \frac{1}{1}$	$A_1 \frac{128}{81}$
Mathna III	$B_1 \frac{16}{9}$	$(B_1) \frac{91}{48}$	$C_1 \frac{52}{27}$	$C \frac{1}{1}$	$O \frac{256}{243}$	$O_1 \frac{13}{12}$	$O \frac{9}{8}$	$B_1 \frac{32}{27}$
Mathlath IV	$F \frac{4}{3}$	$(F_1) \frac{91}{64}$	$G_1 \frac{13}{9}$	$G \frac{3}{2}$	$A_1 \frac{27}{16}$	$A_1 \frac{13}{8}$	$A \frac{27}{16}$	$B_1 \frac{16}{9}$
Bamm V	$C \frac{1}{1}$	$(C_1) \frac{273}{256}$	$O_1 \frac{13}{12}$	$O \frac{9}{8}$	$B_1 \frac{32}{27}$	$E_1 \frac{39}{32}$	$E \frac{81}{64}$	$F \frac{4}{3}$

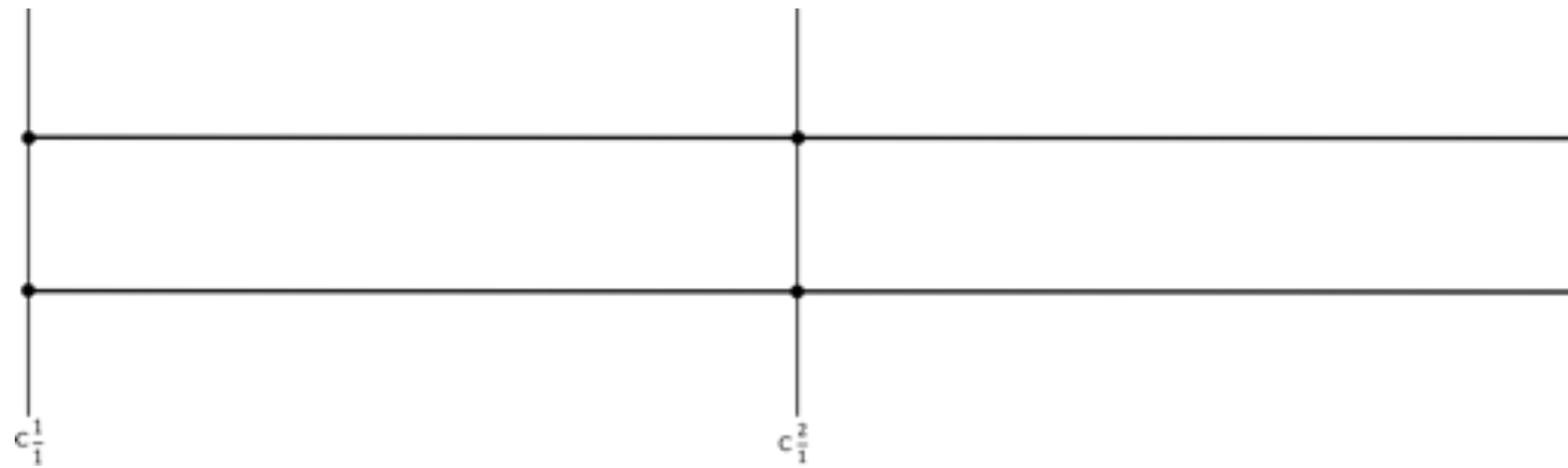
- the first and second octave are not identical
- 13-Limit neutral Intervals (quartertunes)
- Using of limma and 273/256 which is approximately a apotome

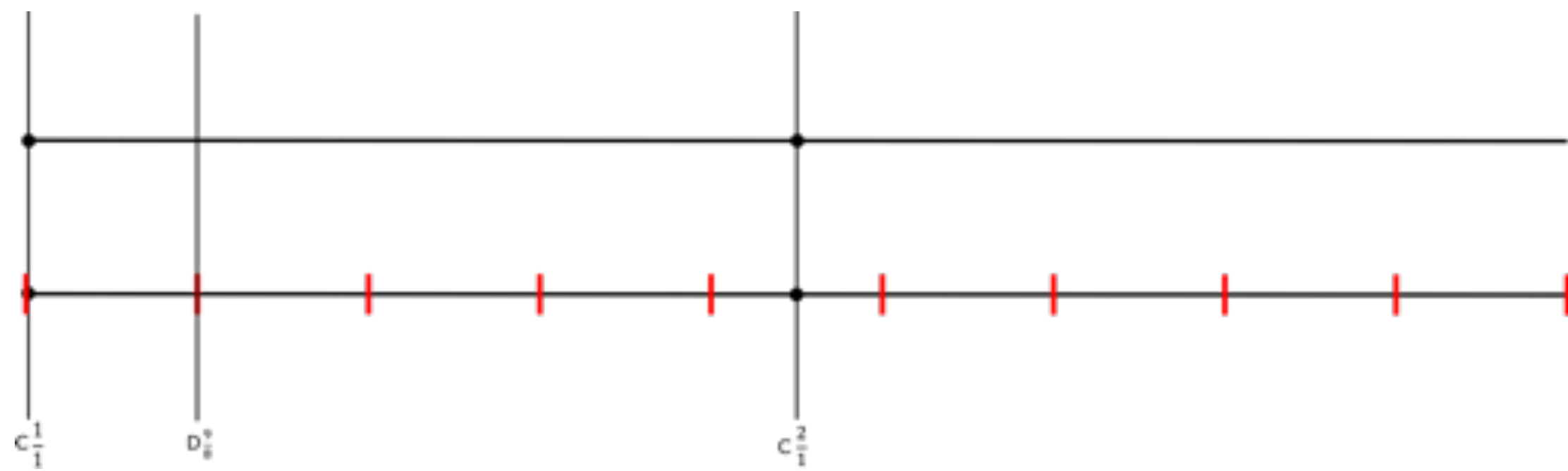


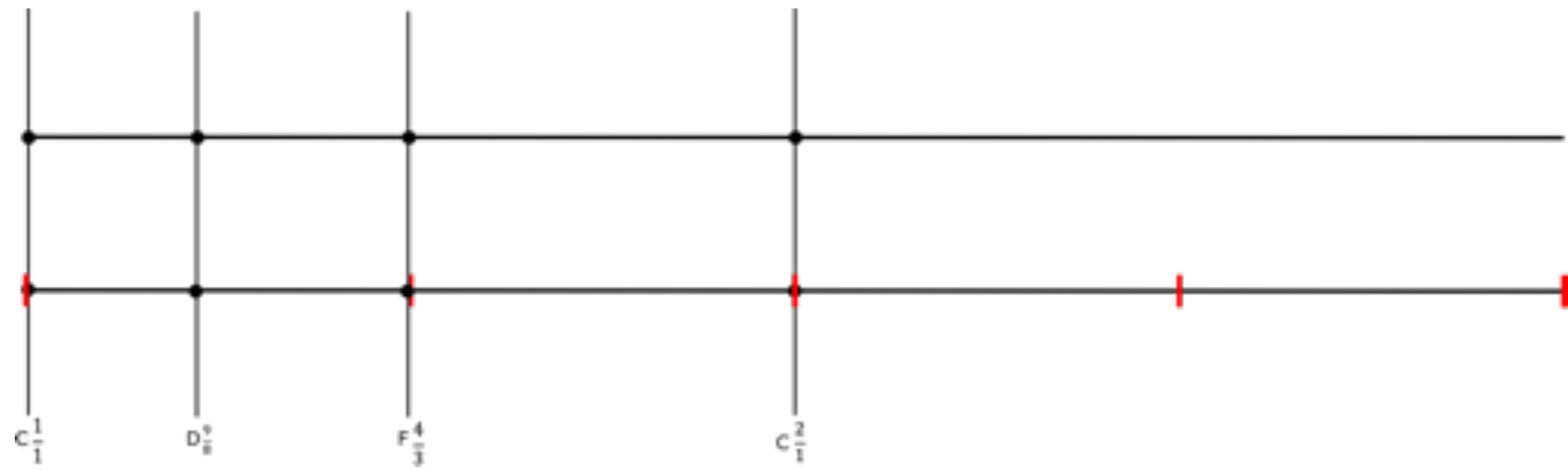
Notation	Ratio	Cent	Notation	Ratio	Cent
C	1/1	0	G $\flat$ '	1024/729	588
C $\sharp$	273/256	111	D $\flat$ '	256/243	90
D $\flat$	13/12	139	D $\flat$ '	13/12	139
D	9/8	204	D'	9/8	204
E $\flat$	32/27	294	E $\flat$ '	32/27	294
E $\flat$	39/32	342	E'	81/64	408
E	81/64	408	F $\flat$ '	104/81	433
F	4/3	498	F'	4/3	498
F $\sharp$	91/64	609	G $\flat$ '	1024/729	588
G $\flat$	13/9	637	G $\flat$ '	13/9	637
G	3/2	702	G'	3/2	702
A $\flat$	128/81	792	A $\flat$ '	128/81	792
A $\flat$	13/8	841	A'	27/16	906
A	27/16	906	A $\sharp$ '	416/243	931
B $\flat$	16/9	996	B $\flat$ '	16/9	996
B	91/48	1107	C $\flat$ '	221/118	1086
C $\flat$	52/27	1135	C $\flat$ '	52/27	1135
C	2/1	1200	C	2/1	1200
			D $\flat$ ''	256/243	90

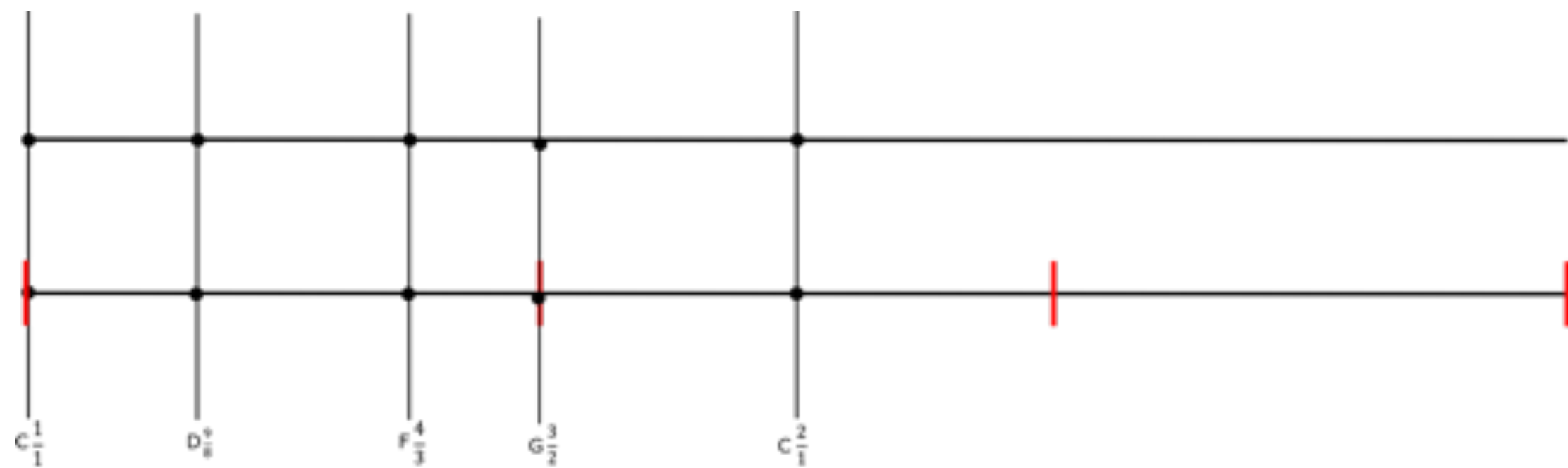
Tuning System of Tanbur of Khorasan as calculated by Farabi

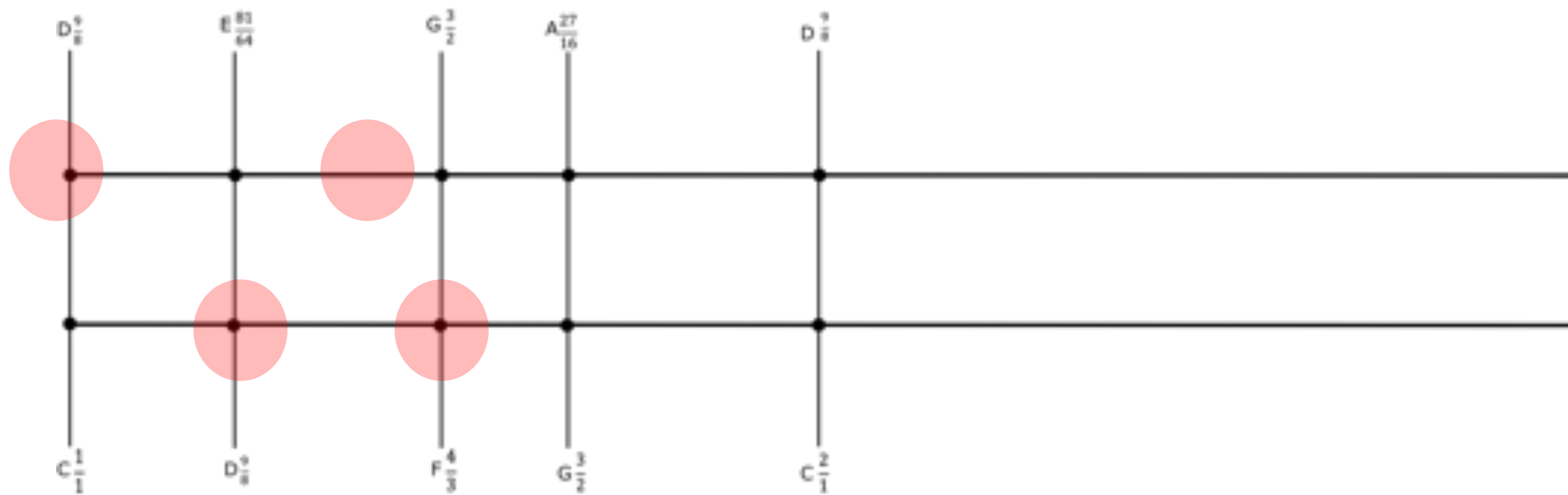


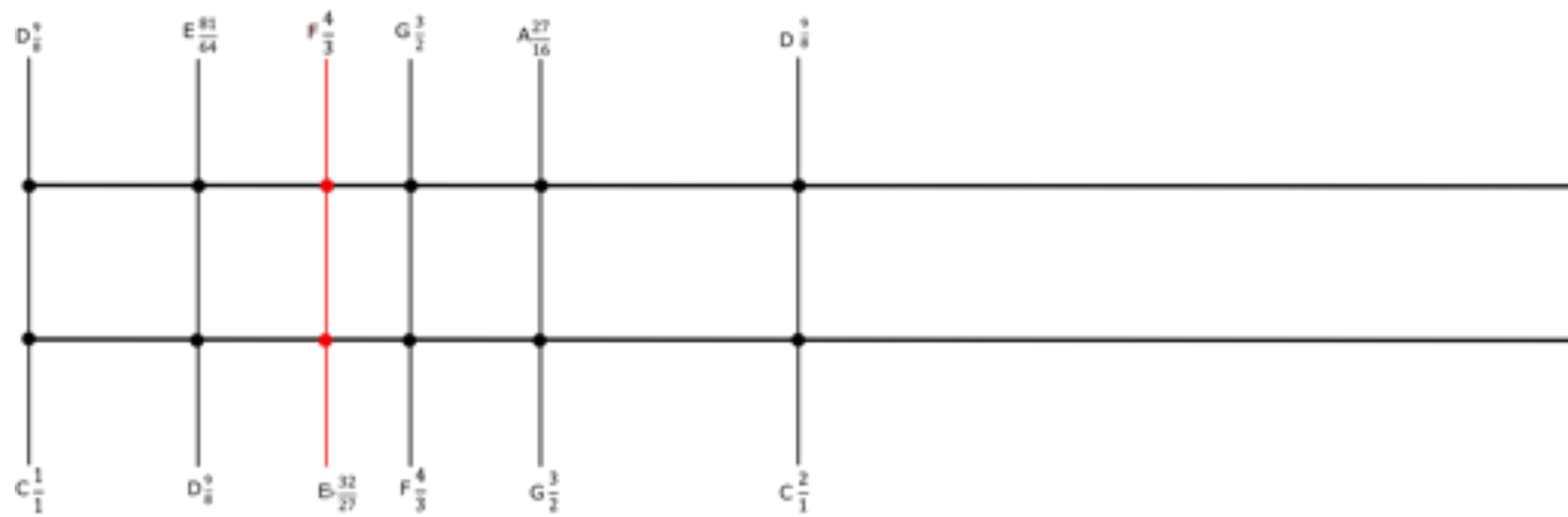


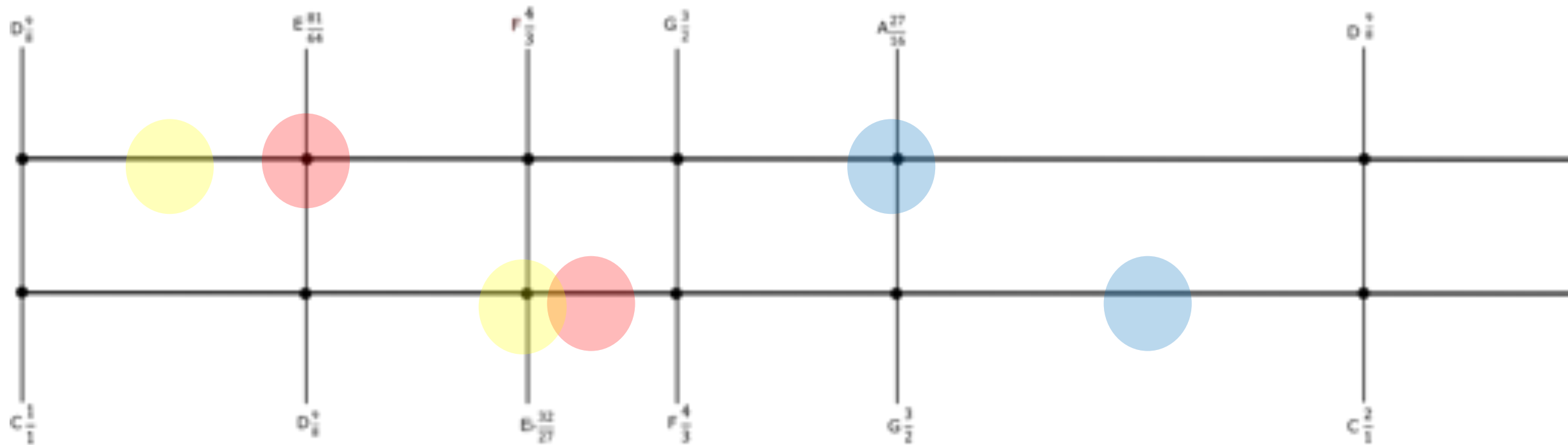


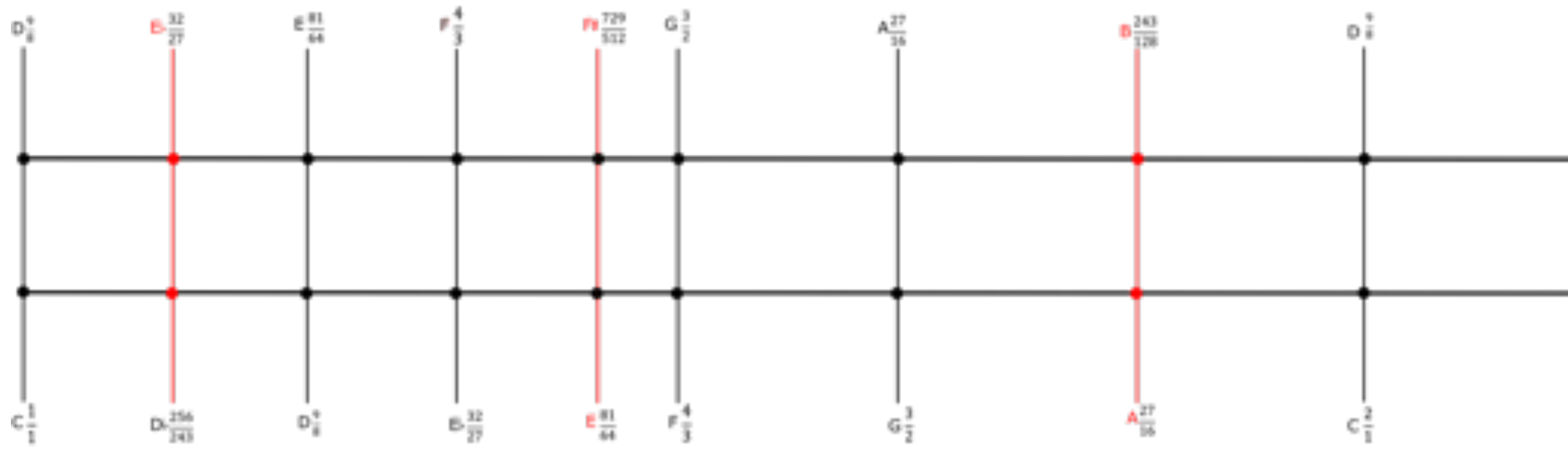


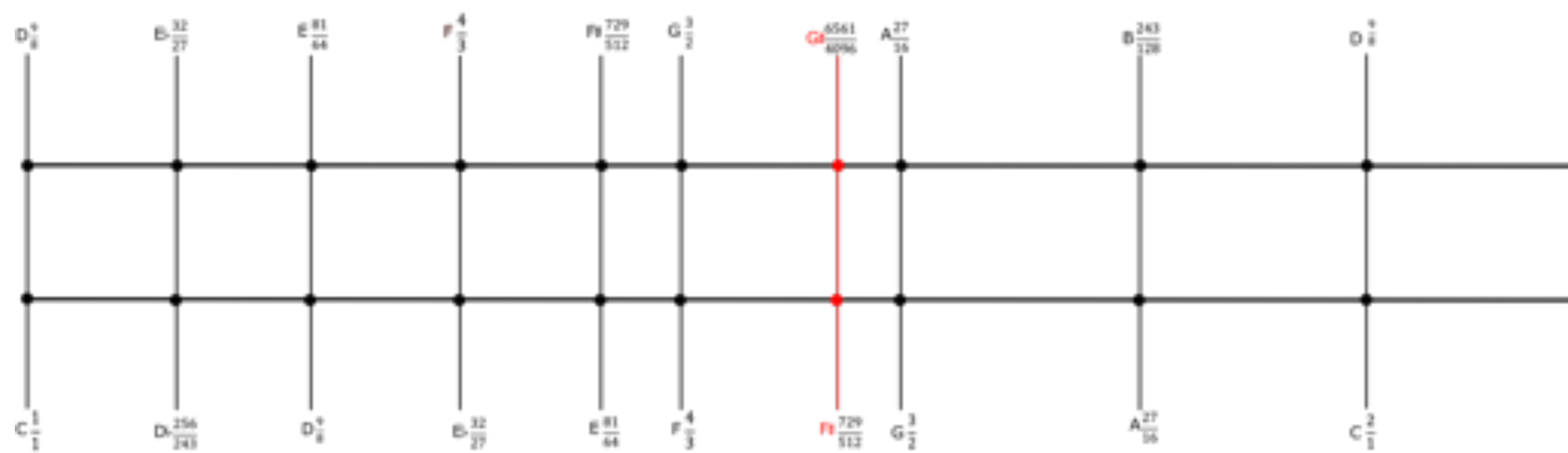


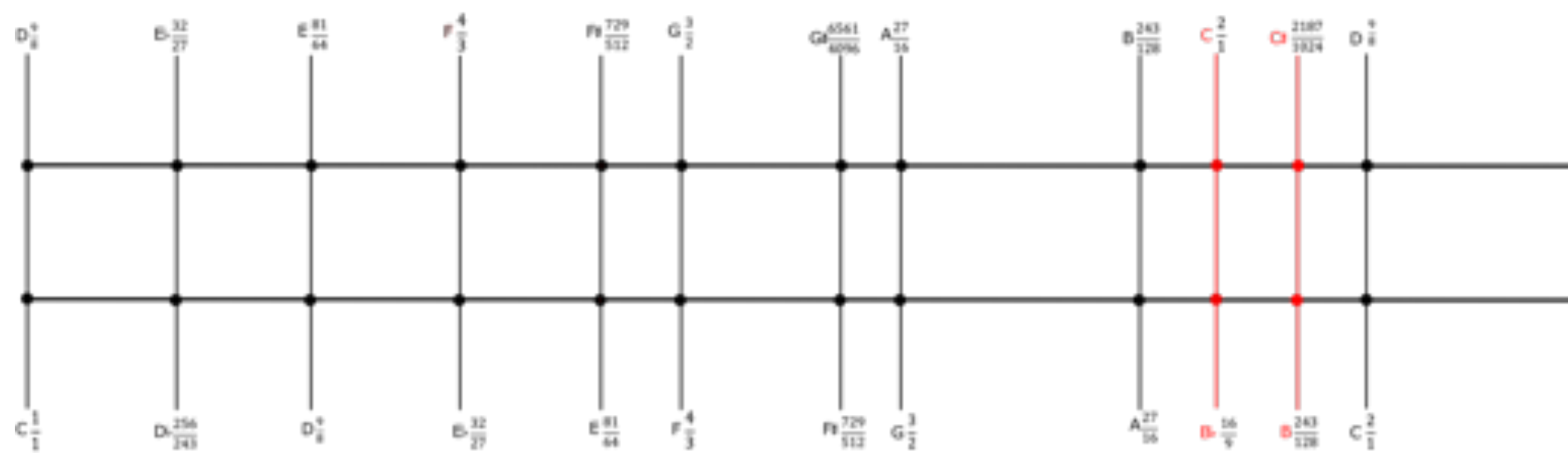


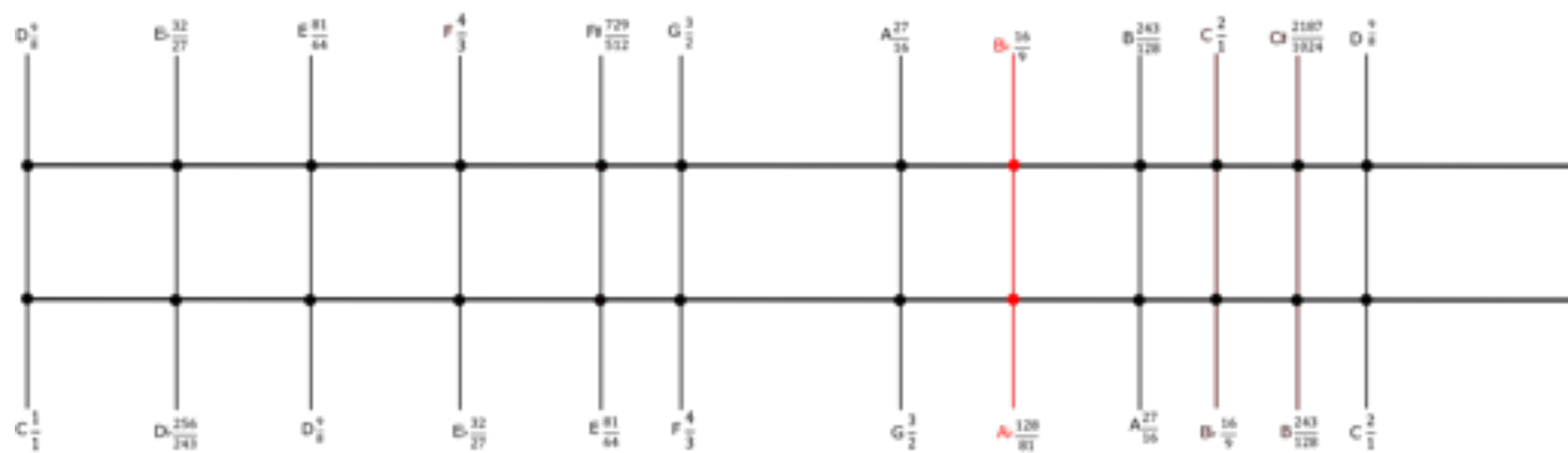


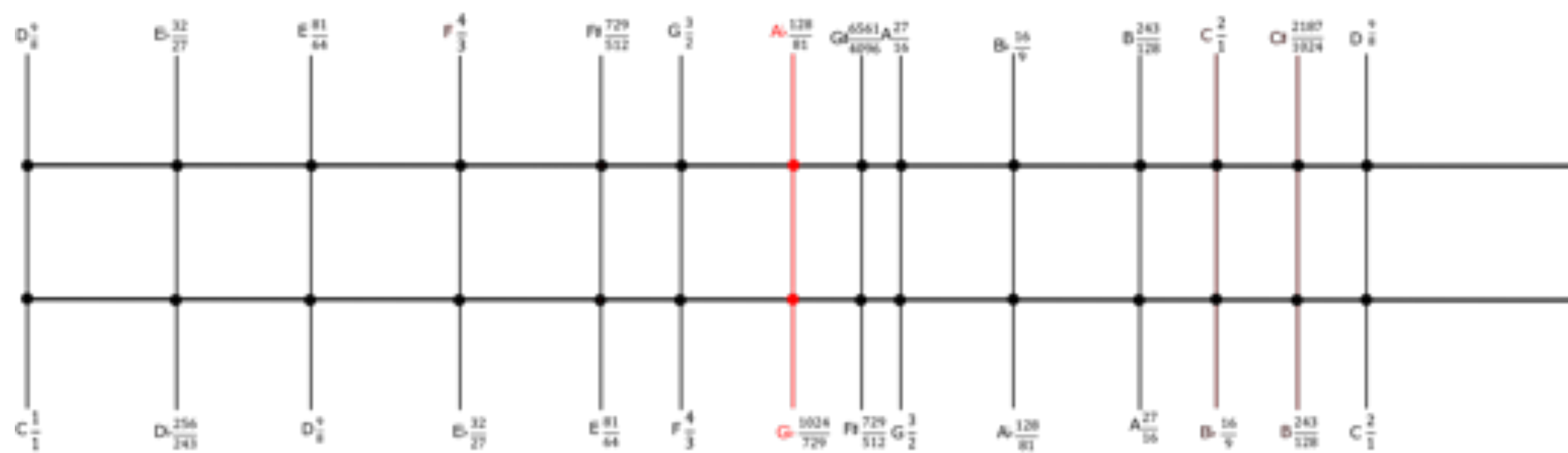


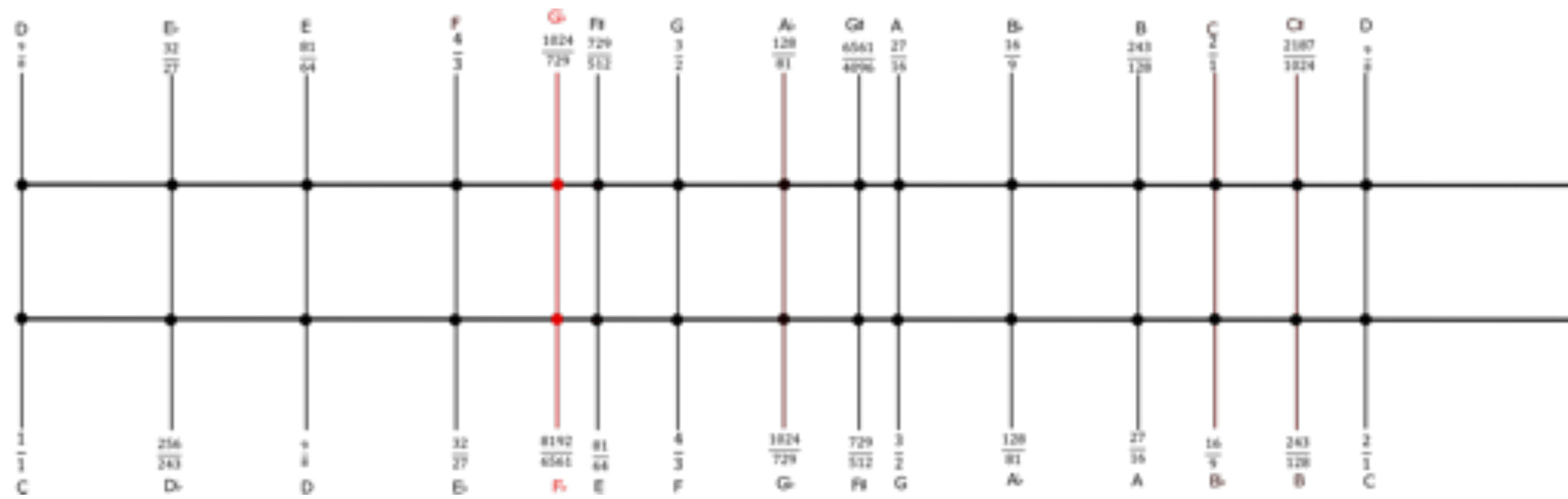


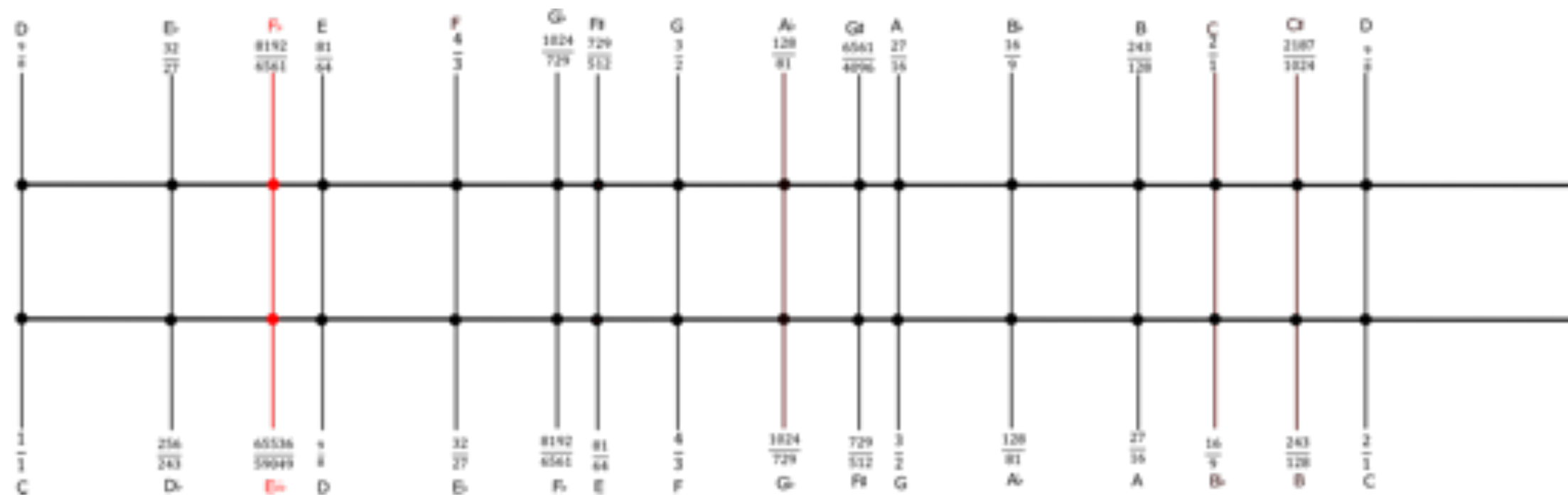


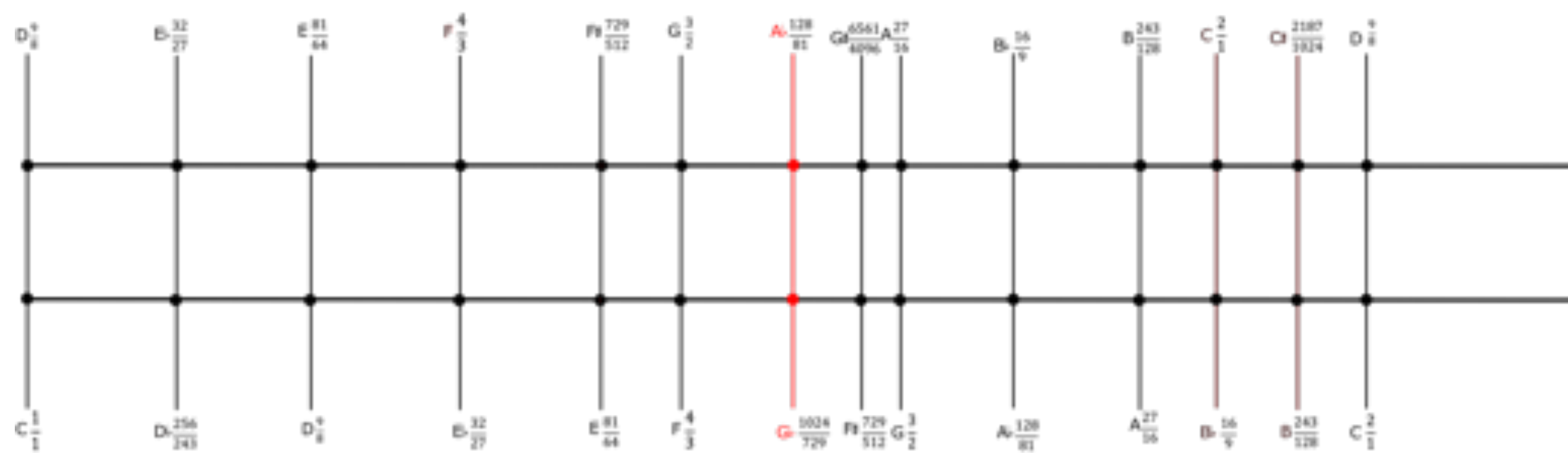


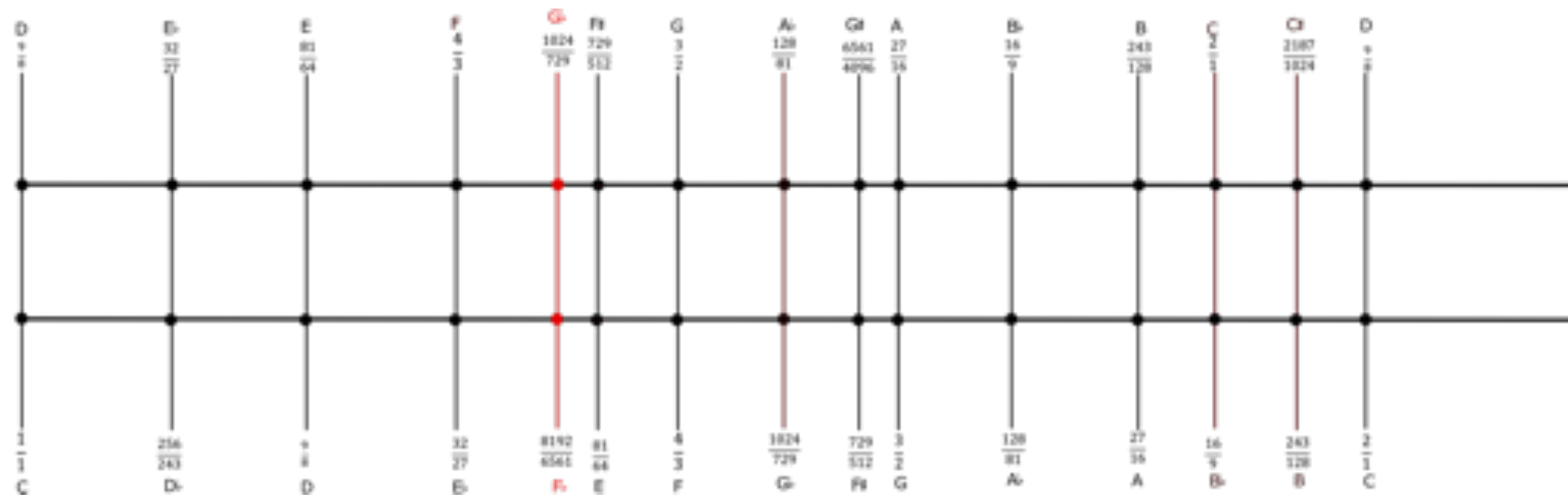


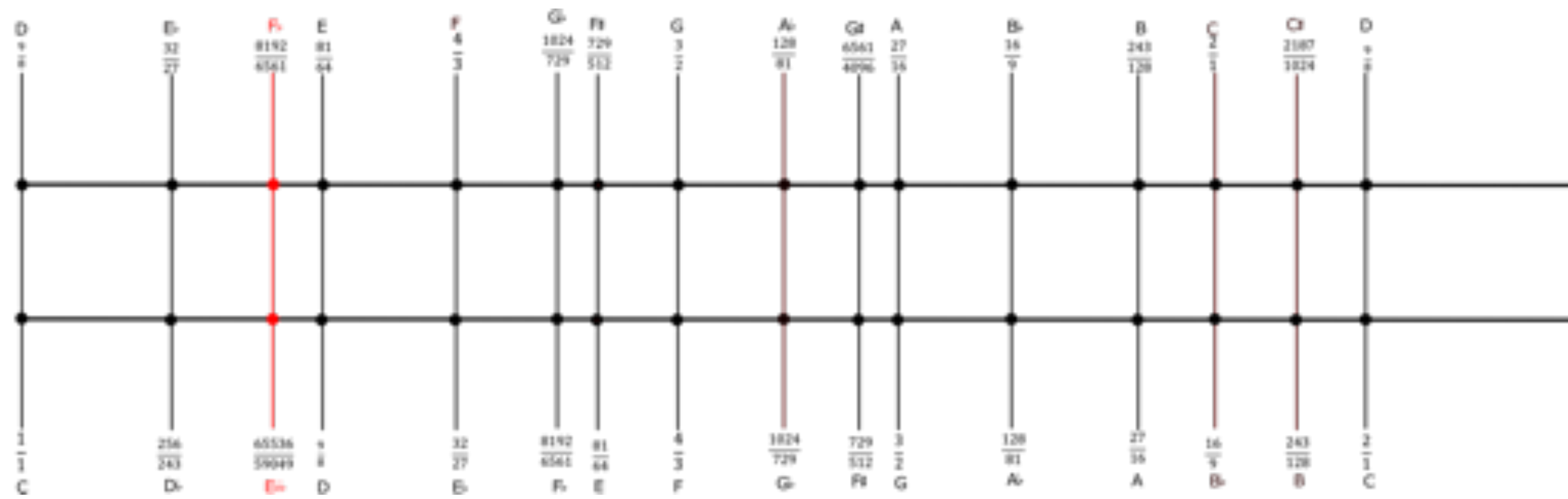


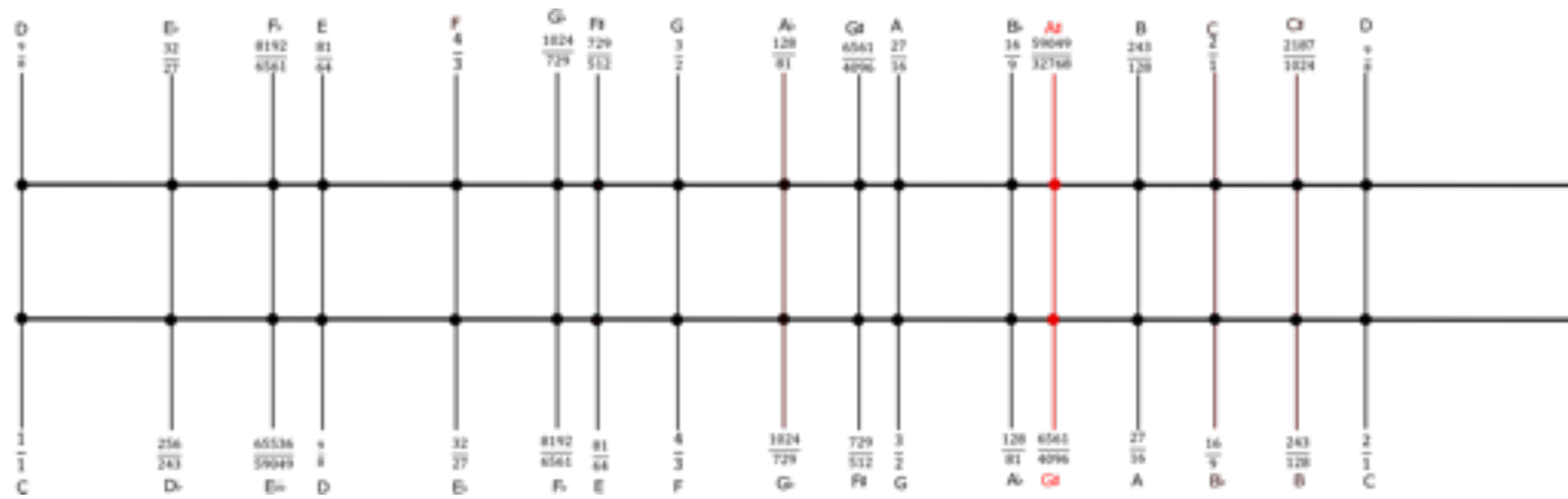


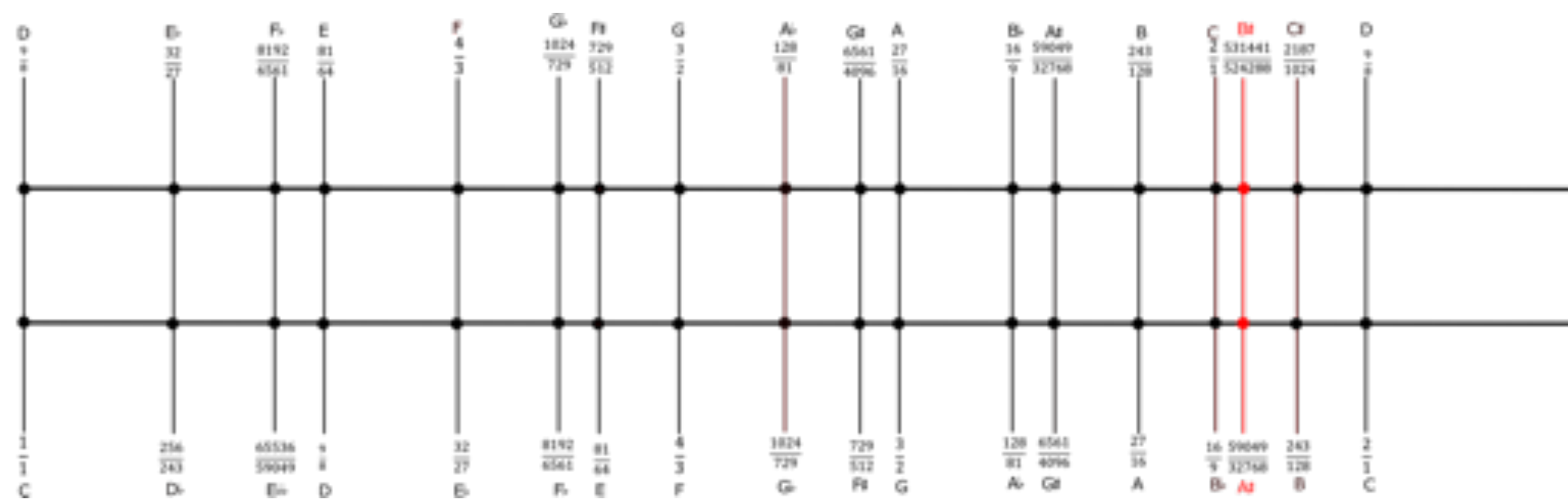


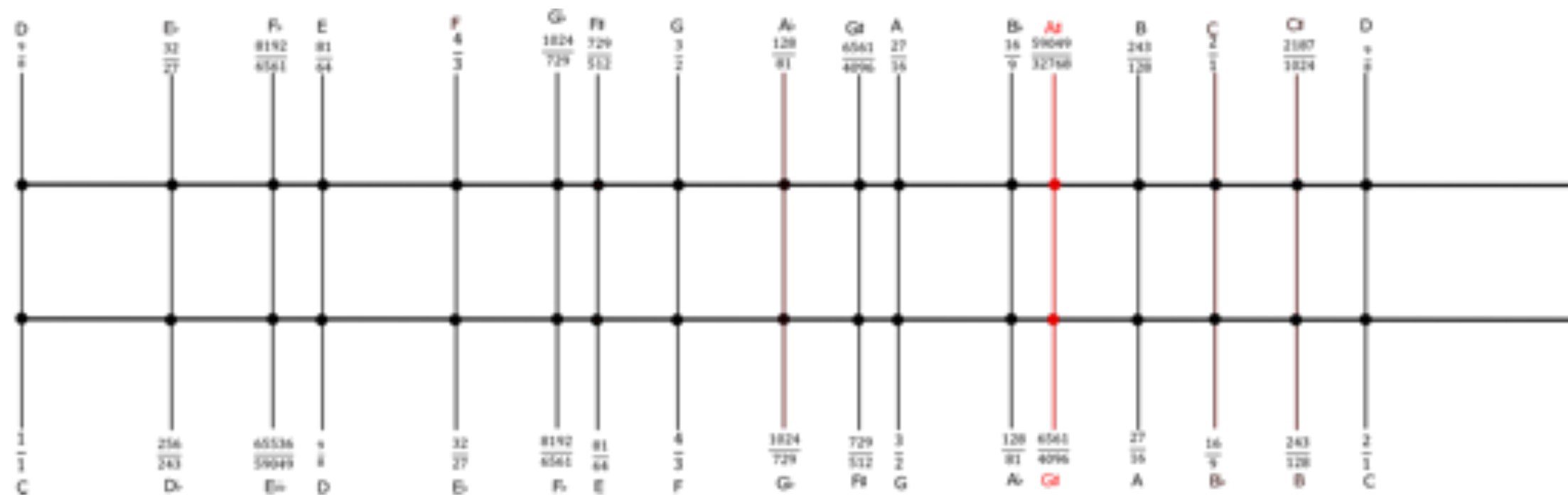


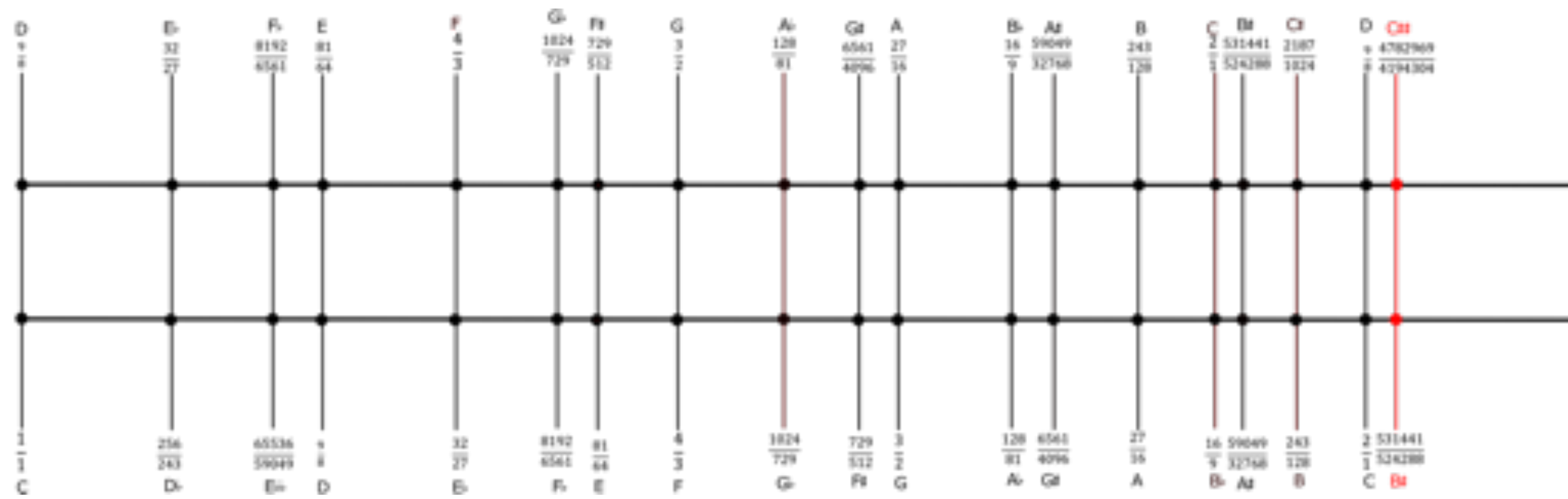


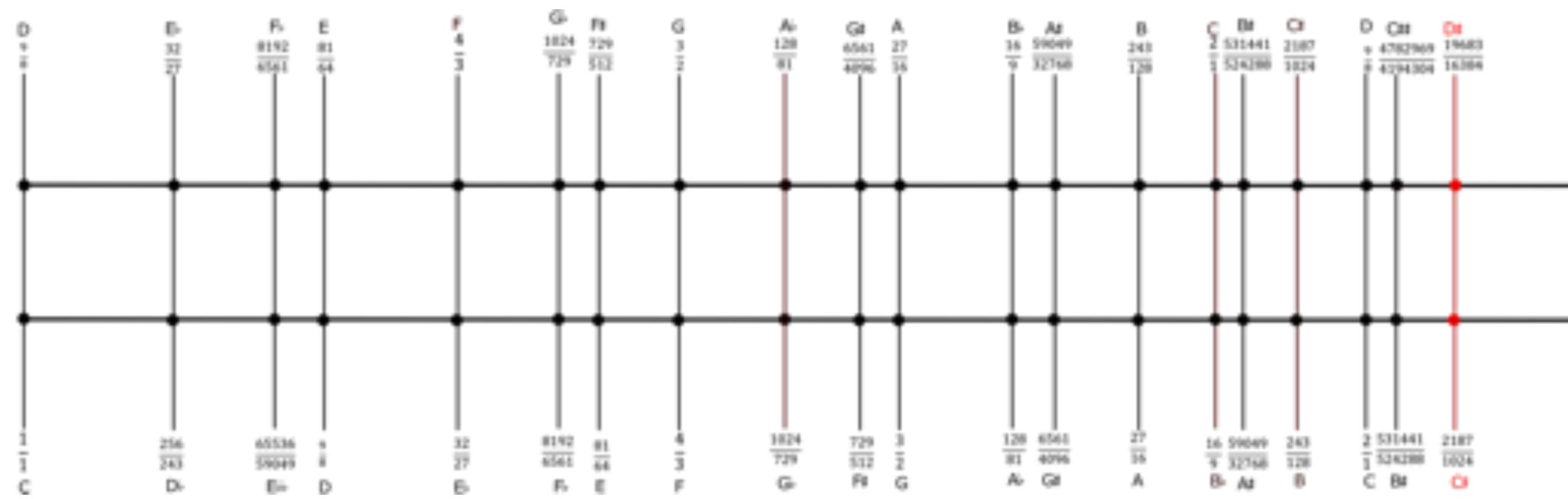


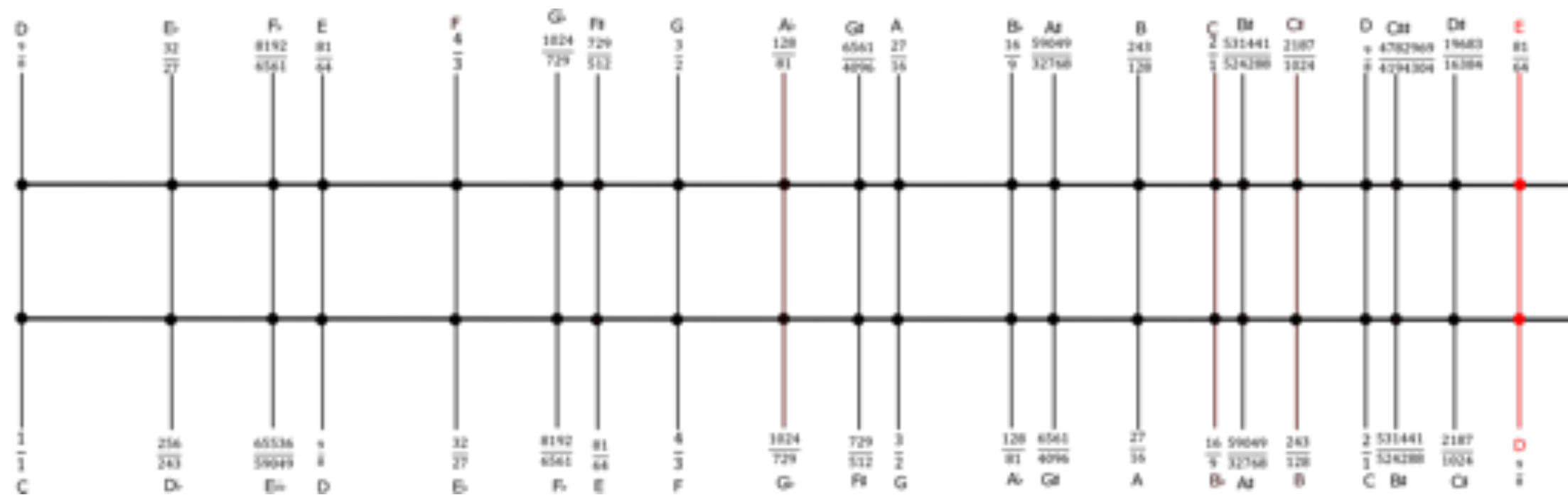


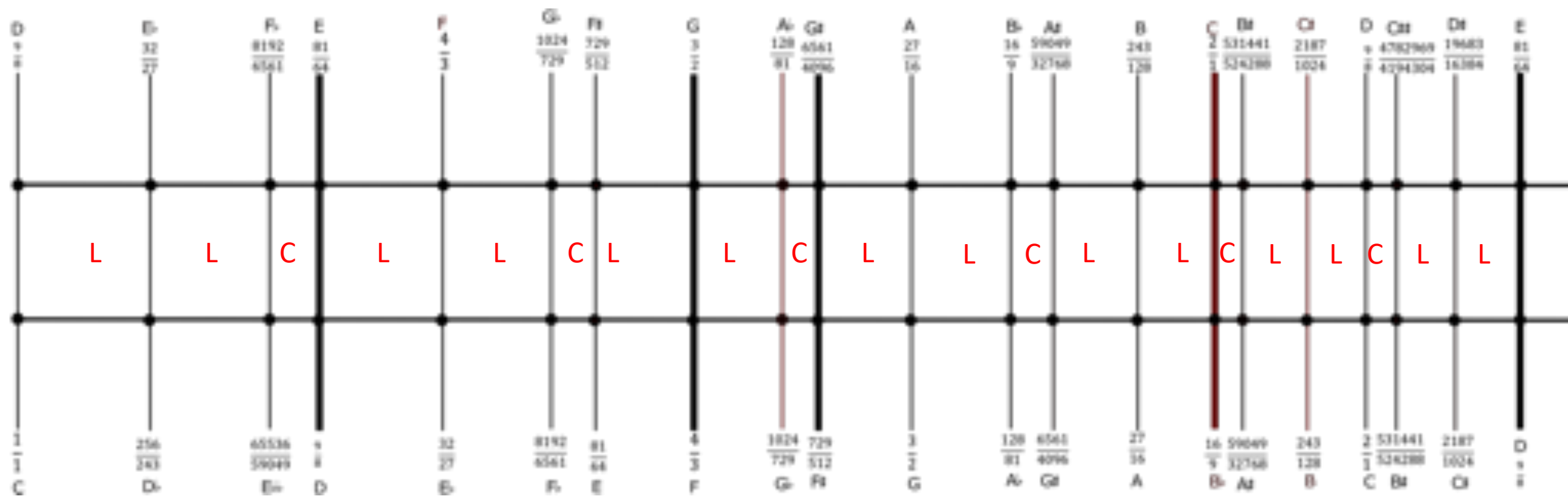




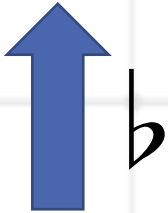






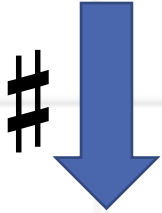


Notation	Ratio	Cent
A $\flat\flat$	$\frac{2^{18}}{3^{11}} = \frac{262144}{177147}$	678.50
E $\flat\flat$	$\frac{2^{16}}{3^{10}} = \frac{65536}{59049}$	180.45
B $\flat\flat$	$\frac{2^{15}}{3^9} = \frac{32768}{19683}$	882.41
F $\flat$	$\frac{2^{13}}{3^8} = \frac{8192}{6561}$	384.36
C $\flat$	$\frac{2^{11}}{3^7} = \frac{2048}{2187}$	1086.31
G $\flat$	$\frac{2^{10}}{3^6} = \frac{1024}{729}$	588.27
D $\flat$	$\frac{2^8}{3^5} = \frac{256}{243}$	90.23
A $\flat$	$\frac{2^7}{3^4} = \frac{128}{81}$	792.18
E $\flat$	$\frac{2^5}{3^3} = \frac{32}{27}$	294.14
B $\flat$	$\frac{2^4}{3^2} = \frac{16}{9}$	996.09
F	$\frac{2^2}{3^1} = \frac{4}{3}$	498.05
C	$\frac{1}{1}$	0.00



Tuning system of Tanbur of Khorasan

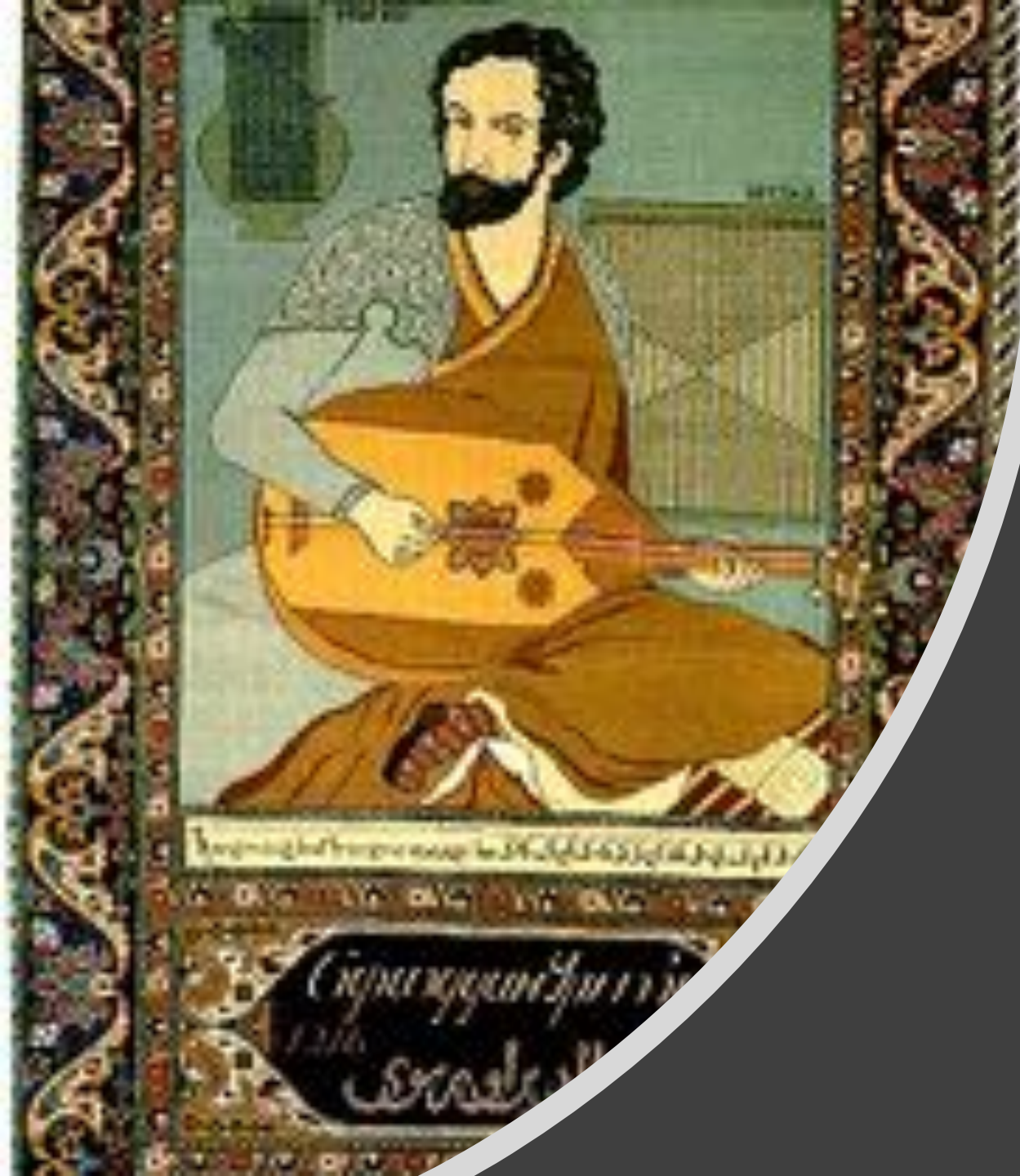
An extended pythagorean tuning system



Notation	Ratio	Cent
C	$\frac{1}{1}$	0.00
G	$\frac{3^1}{2^1} = \frac{3}{2}$	701.96
D	$\frac{3^2}{2^3} = \frac{9}{8}$	203.91
A	$\frac{3^3}{2^4} = \frac{27}{16}$	905.87
E	$\frac{3^4}{2^6} = \frac{81}{64}$	407.82
B	$\frac{3^5}{2^7} = \frac{243}{128}$	1109.78
F $\sharp$	$\frac{3^6}{2^9} = \frac{729}{512}$	611.73
C $\sharp$	$\frac{3^7}{2^{11}} = \frac{2187}{2048}$	113.69
G $\sharp$	$\frac{3^8}{2^{12}} = \frac{6561}{4096}$	815.64
D $\sharp$	$\frac{3^9}{2^{14}} = \frac{19683}{16384}$	317.60
A $\sharp$	$\frac{3^{10}}{2^{15}} = \frac{59049}{32768}$	1019.56
E $\sharp$	$\frac{3^{11}}{2^{17}} = \frac{177147}{131072}$	521.51
B $\sharp$	$\frac{3^{12}}{2^{18}} = \frac{531441}{262144}$	1223.46

Tuning System of Safi Al-Din Urmawi 1216 - 1293

- Kitab Al-Adwar
- Al-Risala Al-Sharafiyya



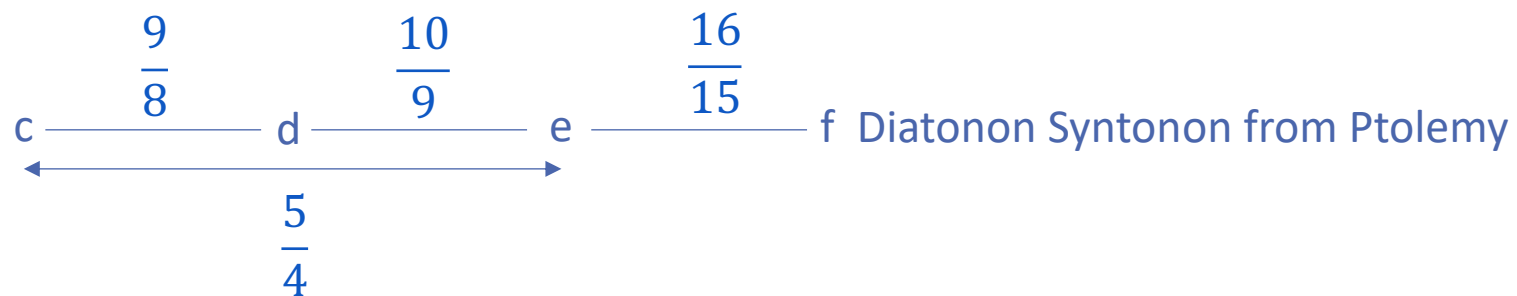
	Khinsir	Binsir	Zatral Wusta	Wusta	Sababab	Mujannab	Zaid	Motlaq
Hal	$\frac{2^6}{243}$	$\frac{1}{2}$	$\frac{2^{12}}{19683}$	$\frac{2^{12}}{19683}$	$\frac{2^4}{9}$	$\frac{2^{22}}{131223}$	$\frac{2^8}{19683}$	$\frac{2^8}{81}$
Zir	$\frac{2^7}{3^4}$	$\frac{3}{2}$	$\frac{2^{14}}{177147}$	$\frac{2^{14}}{729}$	$\frac{2^5}{3}$	$\frac{2^{23}}{109023}$	$\frac{2^{13}}{6561}$	$\frac{2^{13}}{27}$
Mathru	$\frac{2^5}{3^3}$	$\frac{3^2}{8}$	$\frac{2^{14}}{59049}$	$\frac{2^6}{243}$	$\frac{1}{1}$	$\frac{2^{14}}{331441}$	$\frac{2^{11}}{19683}$	$\frac{2^{11}}{9}$
Mathlam	$\frac{2^4}{3^2}$	$\frac{3^3}{16}$	$\frac{2^{13}}{19683}$	$\frac{2^7}{81}$	$\frac{3}{2}$	$\frac{2^{14}}{177147}$	$\frac{2^{12}}{729}$	$\frac{2^{12}}{3}$
Bamir	$\frac{2^5}{108}$	$\frac{5}{15}$	$\frac{2^{13}}{6561}$	$\frac{2^5}{27}$	$\frac{3^2}{8}$	$\frac{2^{14}}{59049}$	$\frac{2^8}{243}$	$\frac{1}{1}$

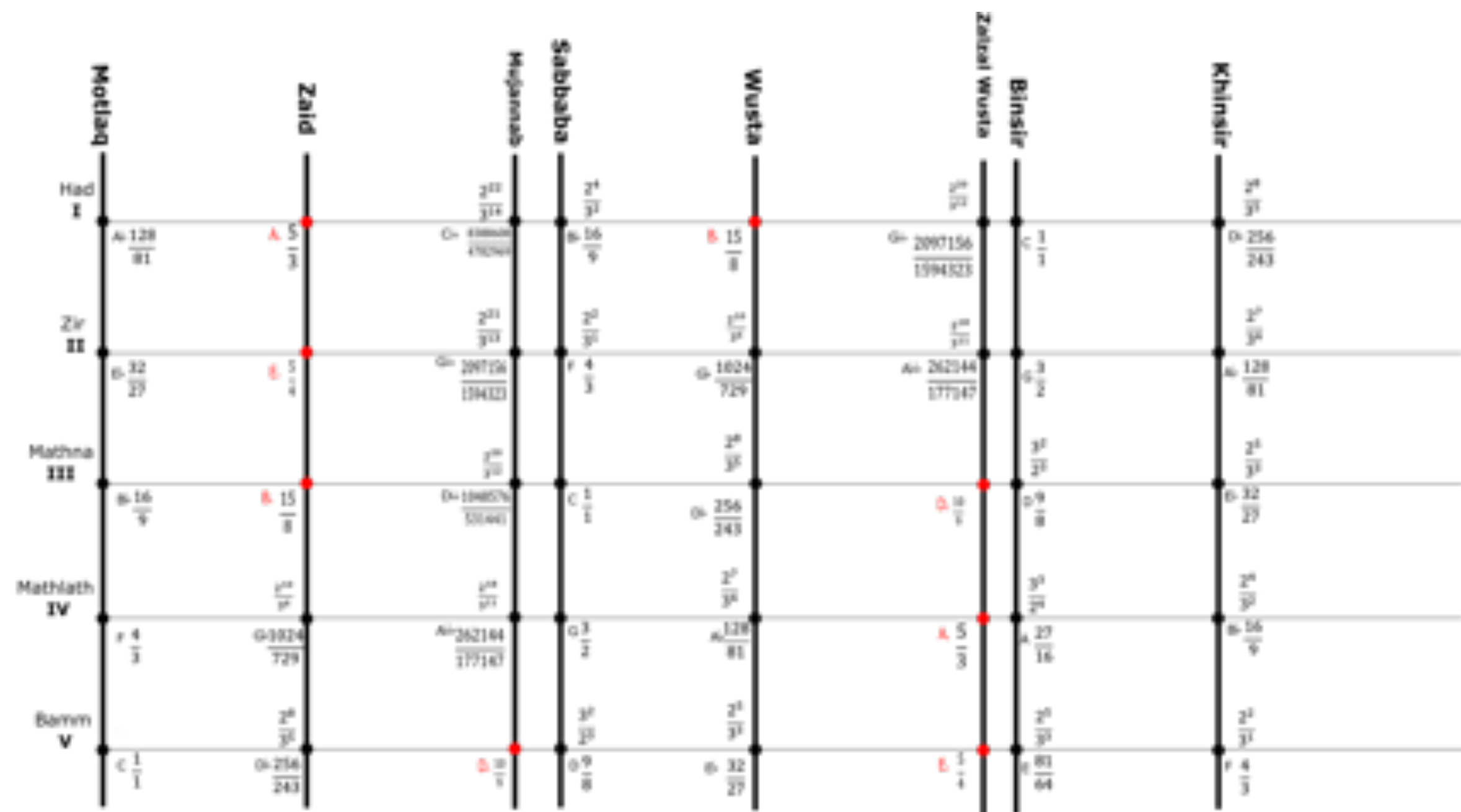
	Motlaq	Zaid	Mujanab	Sabbaba	Wusta	Zalzal Wusta	Binsir	Khinsir
Had I	$A = \frac{128}{81}$	$B = \frac{32768}{19683}$	$C = \frac{8388608}{4782969}$	$B = \frac{16}{9}$	$C = \frac{32768}{19683}$	$G = \frac{2097156}{1594323}$	$C = \frac{1}{1}$	$D = \frac{256}{243}$
Zir II	$C = \frac{32}{27}$	$F = \frac{8192}{6561}$	$G = \frac{2097156}{1594323}$	$F = \frac{4}{3}$	$C = \frac{1024}{729}$	$A = \frac{262144}{177147}$	$G = \frac{3}{2}$	$A = \frac{128}{81}$
Mathna III	$B = \frac{16}{9}$	$C = \frac{32768}{19683}$	$D = \frac{1048576}{531441}$	$C = \frac{1}{1}$	$D = \frac{256}{243}$	$B = \frac{65536}{59049}$	$D = \frac{9}{8}$	$C = \frac{32}{27}$
Mathlath IV	$F = \frac{4}{3}$	$G = \frac{1024}{729}$	$A = \frac{262144}{177147}$	$G = \frac{3}{2}$	$A = \frac{128}{81}$	$B = \frac{32768}{19683}$	$A = \frac{27}{16}$	$B = \frac{16}{9}$
Bamm V	$C = \frac{1}{1}$	$D = \frac{256}{243}$	$B = \frac{65536}{59049}$	$D = \frac{9}{8}$	$B = \frac{32}{27}$	$A = \frac{8192}{6561}$	$E = \frac{81}{64}$	$F = \frac{4}{3}$

$$\frac{8192}{6561} \xrightarrow{\text{Cent}} 384.36 \text{ cent}$$

$$\frac{5}{4} \xrightarrow{\text{Cent}} 386.31 \text{ cent}$$

$$\frac{5}{4} \div \frac{8192}{6561} = \frac{32805}{32768} \xrightarrow{\text{Cent}} 1.953720788 \text{ cent (Shisma Comma)}$$

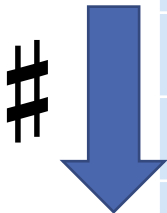
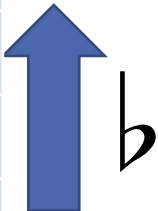




Tuning system of Ormawi

An extended pythagorean tuning system

Notation	Ratio	Cent
Cbb	$\frac{2^{23}}{3^{14}} = \frac{8388608}{4782969}$	1176.54
Gbb	$\frac{2^{21}}{3^{13}} = \frac{2097152}{1594323}$	1176.54
Dbb	$\frac{2^{20}}{3^{12}} = \frac{1048576}{531441}$	1176.54
Abb	$\frac{2^{18}}{3^{11}} = \frac{262144}{177147}$	678.50
Ds	$\frac{10}{9}$	182.40
As	$\frac{5}{3}$	884.36
Es	$\frac{5}{4}$	386.31
Bs	$\frac{15}{8}$	1088.29
Gb	$\frac{2^{10}}{3^6} = \frac{1024}{729}$	588.27
Db	$\frac{2^8}{3^5} = \frac{256}{243}$	90.23
Ab	$\frac{2^7}{3^4} = \frac{128}{81}$	792.18
Eb	$\frac{2^5}{3^3} = \frac{32}{27}$	294.14
Bb	$\frac{2^4}{3^2} = \frac{16}{9}$	996.09
F		498.05
C		0.00



Notation	Ratio	Cent
C	$\frac{1}{1}$	0.00
G	$\frac{3^1}{2^1} = \frac{3}{2}$	701.96
D	$\frac{3^2}{2^3} = \frac{9}{8}$	203.91
A	$\frac{3^3}{2^4} = \frac{27}{16}$	905.87
E	$\frac{3^4}{2^6} = \frac{81}{64}$	407.82



	Motlaq	Zaid 1	Mujannab 01	Sabbaba	Wusta	Zalzal Wuster	Binsir	Khinsir
Had I	$A = \frac{128}{81}$	$\frac{32768}{19683}$		$B = \frac{16}{9}$	$C = \frac{32768}{19683}$	$C = \frac{64}{33}$	$C = \frac{1}{1}$	$D = \frac{256}{243}$
Zir II	$B = \frac{32}{27}$	$F = \frac{8192}{6561}$	$F = \frac{192}{149}$	$F = \frac{4}{3}$	$G = \frac{1024}{729}$	$G = \frac{16}{11}$	$G = \frac{8}{2}$	$A = \frac{128}{81}$
Mathna III	$B = \frac{16}{9}$	$\frac{32768}{19683}$	$C = \frac{288}{149}$	$C = \frac{4}{3}$	$D = \frac{256}{243}$	$D = \frac{12}{11}$	$D = \frac{9}{8}$	$B = \frac{32}{27}$
Mathlath IV	$F = \frac{4}{3}$	$G = \frac{1024}{729}$	$G = \frac{216}{149}$	$G = \frac{3}{2}$	$A = \frac{27}{16}$	$A = \frac{18}{11}$	$A = \frac{27}{16}$	$B = \frac{16}{9}$
Bamn V	$C = \frac{1}{1}$	$D = \frac{256}{243}$	$D = \frac{162}{149}$	$D = \frac{9}{8}$	$B = \frac{32}{27}$	$E = \frac{27}{22}$	$E = \frac{81}{64}$	$F = \frac{4}{3}$

	Motlaq	Zaid 1	Mujannab 01	Sabbaba	Wusta	Zaizal Wuster	Binsir	Khinsir
Had I	$A = \frac{128}{81}$	$\frac{32768}{19683}$		$B = \frac{16}{9}$	$C = \frac{32768}{19683}$	$D = \frac{64}{33}$	$C = \frac{1}{11}$	$D = \frac{256}{243}$
Zir II	$B = \frac{32}{27}$	$F = \frac{8192}{6561}$	$F = \frac{192}{149}$	$F = \frac{4}{33}$	$G = \frac{1024}{729}$	$G = \frac{16}{11}$	$G = \frac{3}{22}$	$A = \frac{128}{81}$
Mathna III	$B = \frac{16}{9}$	$\frac{32768}{19683}$	$C = \frac{288}{149}$	$C = \frac{1}{11}$	$D = \frac{256}{243}$	$D = \frac{12}{11}$	$D = \frac{9}{8}$	$B = \frac{32}{27}$
Mathlath IV	$F = \frac{4}{3}$	$G = \frac{1024}{729}$	$G = \frac{216}{149}$	$G = \frac{3}{22}$	$A = \frac{27}{16}$	$A = \frac{18}{11}$	$A = \frac{27}{16}$	$B = \frac{16}{9}$
Bamm V	$C = \frac{1}{11}$	$D = \frac{256}{243}$	$D = \frac{162}{149}$	$D = \frac{9}{8}$	$B = \frac{32}{27}$	$E = \frac{27}{22}$	$E = \frac{81}{64}$	$F = \frac{4}{3}$

	Motlaq	Zaid	Mujannab	Sabbaba	Wusta	Zalzal Wustar (persian Wusta)	Binsir	Khinsir
Had I	$A_1 \frac{128}{81}$	$A_1 \frac{5}{3}$	$A_1 \frac{65536}{38115}$	$B_1 \frac{16}{9}$	$B_1 \frac{15}{8}$	$C_1 \frac{1024}{531}$	$C_1 \frac{1}{1}$	$D_1 \frac{256}{243}$
Zir II	$B_2 \frac{32}{27}$	$E_1 \frac{5}{6}$	$F_1 \frac{16384}{12765}$	$F_1 \frac{4}{3}$	$G_1 \frac{1024}{729}$	$G_1 \frac{256}{177}$	$G_1 \frac{3}{2}$	$H_1 \frac{128}{81}$
Mathna III	$B_2 \frac{16}{9}$	$B_1 \frac{15}{8}$	$C_1 \frac{8192}{4235}$	$C_1 \frac{1}{1}$	$D_1 \frac{256}{243}$	$D_1 \frac{64}{59}$	$D_1 \frac{9}{8}$	$E_1 \frac{32}{27}$
Mathlath IV	$F_1 \frac{4}{3}$	$G_1 \frac{1024}{729}$	$G_1 \frac{6144}{4235}$	$G_1 \frac{3}{2}$	$A_1 \frac{27}{16}$	$A_1 \frac{96}{59}$	$A_1 \frac{27}{16}$	$B_1 \frac{16}{9}$
Bamm V	$C_1 \frac{1}{1}$	$D_1 \frac{256}{243}$	$D_1 \frac{4608}{4235}$	$D_1 \frac{9}{8}$	$B_1 \frac{32}{27}$	$E_1 \frac{72}{59}$	$E_1 \frac{81}{64}$	$F_1 \frac{4}{3}$

	Motlaq	Zaid	Mujannab	Sabbaba	Wusta	Zalzal Wuster (persian Wusta)	Binsir	Khinsir
Had I	$A_0 \frac{128}{81}$	$A_1 \frac{5}{3}$	$A_2 \frac{65536}{38115}$	$B_0 \frac{16}{9}$	$B_1 \frac{15}{8}$	$C_1 \frac{1024}{531}$	$C_0 \frac{1}{1}$	$D_0 \frac{256}{243}$
Zir II	$B_0 \frac{32}{27}$	$E_1 \frac{5}{4}$	$F_1 \frac{16384}{12765}$	$F_0 \frac{4}{3}$	$G_0 \frac{1024}{729}$	$G_1 \frac{256}{177}$	$G_2 \frac{3}{2}$	$A_0 \frac{128}{81}$
Mathna III	$B_0 \frac{16}{9}$	$B_1 \frac{15}{8}$	$C_1 \frac{8192}{4235}$	$C_0 \frac{1}{1}$	$D_0 \frac{256}{243}$	$D_1 \frac{64}{59}$	$D_2 \frac{9}{8}$	$B_0 \frac{32}{27}$
Mathlath IV	$F_0 \frac{4}{3}$	$G_0 \frac{1024}{729}$	$G_1 \frac{6144}{4235}$	$G_2 \frac{3}{2}$	$A_0 \frac{27}{16}$	$A_1 \frac{96}{59}$	$A_2 \frac{27}{16}$	$B_0 \frac{16}{9}$
Bamm V	$C_0 \frac{1}{1}$	$D_0 \frac{256}{243}$	$D_1 \frac{4608}{4235}$	$D_2 \frac{9}{8}$	$B_0 \frac{32}{27}$	$E_1 \frac{72}{59}$	$E_0 \frac{81}{64}$	$F_0 \frac{4}{3}$

Notation	Ratio	Cent
C	1/1	0
D $\flat$	256/243	90
D $\natural$	803/738	146
D	9/8	204
E $\flat$	32/27	294
E $\natural$	72/59	345
F $\flat$	221/177	384
E	81/64	408
F	4/3	498
G $\flat$	1024/729	588
G $\natural$	663/457	644
G	3/2	702
A $\flat$	128/81	792
A $\natural$	96/59	843
B $\flat\flat$	32768/19683	882
A	27/16	906
B $\flat$	16/9	996
C $\flat$	1366/729	1087
C $\natural$	32/17	1095
C $\sharp$	1385/716	1142

Notation	Ratio	Cent
C'	2/1	1200
D $\flat$ '	256/243	90
D $\natural$ '	64/59	141
D'	9/8	204
E $\flat$ '	32/27	294
Es'	5/4	386
F $\natural$ '	1051/815	440
F'	4/3	498
G $\flat$ '	1024/729	588
G $\natural$ '	256/177	639
G'	3/2	702
A $\flat$ '	128/81	792
As'	5/3	884
A $\sharp$ '	65536/38115	882
B $\flat$ '	16/9	996
Bs'	15/8	1088
C $\natural$ '	1024/531	1137
C''	4/1	2400
D $\flat$ ''	256/243	90

Iranian Tuning System

Iranian modern Tar and Setar Tuning System (Hormoz Farhat's research)	Notation	Cent	Cent	Ratio	Ibn Sina's Tuning System
	C	0	90	1/1	
	D $\flat$	90	90	256/243	
	D $\natural$	135	139	13/12	
	D	205	204	9/8	
	E $\flat$	295	294	32/27	
	E $\natural$	340	342	39/32	
	E	410	408	81/64	
	F	500	498	4/3	
	F $\sharp$	565	609	91/64	
	G $\natural$	630	637	13/9	
	G	700	702	3/2	
	A $\flat$	790	792	128/81	
	A $\natural$	835	841	13/8	
	A	905	906	27/16	
	B $\flat$	995	996	16/9	
	B $\natural$	1040	1107	91/48	
	B	1110	1135	52/27	
	C	1200	1200	2/1	

Turkish Tuning System

Yekta bey

C	$\frac{1}{1}$	0.00
B#	$\frac{3^{12}}{2^{18}} = \frac{531441}{262144}$	23
Db	$\frac{2^8}{3^5} = \frac{256}{243}$	90
C#	$\frac{3^7}{2^{11}} = \frac{2187}{2048}$	114
Ebb	$\frac{2^{16}}{3^{10}} = \frac{65536}{59049}$	180
D	$\frac{3^2}{2^3} = \frac{9}{8}$	204
Eb	$\frac{2^5}{3^3} = \frac{32}{27}$	294
D#	$\frac{3^9}{2^{14}} = \frac{19683}{16384}$	318
Fb	$\frac{2^{13}}{3^8} = \frac{8192}{6561}$	384
E	$\frac{3^4}{2^6} = \frac{81}{64}$	408
Gbb	$\frac{2^{21}}{3^{13}} = \frac{2097152}{1594323}$	475
F	$\frac{2^2}{3^1} = \frac{4}{3}$	498
E#	$\frac{3^{11}}{2^{17}} = \frac{177147}{131072}$	522
Gb	$\frac{2^{10}}{3^6} = \frac{1024}{729}$	588
F#	$\frac{3^6}{2^9} = \frac{729}{512}$	612
Abb	$\frac{2^{18}}{3^{11}} = \frac{262144}{177147}$	679
G	$\frac{3^1}{2^1} = \frac{3}{2}$	702

Ab	$\frac{2^7}{3^4} = \frac{128}{81}$	792
G#	$\frac{3^8}{2^{12}} = \frac{6561}{4096}$	816
Bbb	$\frac{2^{15}}{3^9} = \frac{32768}{19683}$	882
A	$\frac{3^3}{2^4} = \frac{27}{16}$	906
Cbb	$\frac{2^{23}}{3^{14}} = \frac{8388608}{4782969}$	973
Bb	$\frac{2^4}{3^2} = \frac{16}{9}$	996
A#	$\frac{3^{10}}{2^{15}} = \frac{59049}{32768}$	1020
Cb	$\frac{2^{11}}{3^7} = \frac{2048}{2187}$	1086
B	$\frac{3^5}{2^7} = \frac{243}{128}$	1110
Dbb	$\frac{2^{20}}{3^{12}} = \frac{1048576}{531441}$	1177

C	$\frac{1}{1}$	0.00
B#	$\frac{3^{12}}{2^{18}} = \frac{531441}{262144}$	23
Db	$\frac{2^8}{3^5} = \frac{256}{243}$	90
C#	$\frac{3^7}{2^{11}} = \frac{2187}{2048}$	114
Ebb	$\frac{2^{16}}{3^{10}} = \frac{65536}{59049}$	180
D	$\frac{3^2}{2^3} = \frac{9}{8}$	204
Eb	$\frac{2^5}{3^3} = \frac{32}{27}$	294
D#	$\frac{3^9}{2^{14}} = \frac{19683}{16384}$	318
Fb	$\frac{2^{13}}{3^8} = \frac{8192}{6561}$	384
E	$\frac{3^4}{2^6} = \frac{81}{64}$	408
Gbb	$\frac{2^{21}}{3^{13}} = \frac{2097152}{1594323}$	475
F	$\frac{2^2}{3^1} = \frac{4}{3}$	498
E#	$\frac{3^{11}}{2^{17}} = \frac{177147}{131072}$	522
Gb	$\frac{2^{10}}{3^6} = \frac{1024}{729}$	588
F#	$\frac{3^6}{2^9} = \frac{729}{512}$	612
Abb	$\frac{2^{18}}{3^{11}} = \frac{262144}{177147}$	679
G	$\frac{3^1}{2^1} = \frac{3}{2}$	702

Ab	$\frac{2^7}{3^4} = \frac{128}{81}$	792
G#	$\frac{3^8}{2^{12}} = \frac{6561}{4096}$	816
Bbb	$\frac{2^{15}}{3^9} = \frac{32768}{19683}$	882
A	$\frac{3^3}{2^4} = \frac{27}{16}$	906
Cbb	$\frac{2^{23}}{3^{14}} = \frac{8388608}{4782969}$	973
Bb	$\frac{2^4}{3^2} = \frac{16}{9}$	996
A#	$\frac{3^{10}}{2^{15}} = \frac{59049}{32768}$	1020
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B	$\frac{3^5}{2^7} = \frac{243}{128}$	1110
Dbb	$\frac{2^{20}}{3^{12}} = \frac{1048576}{531441}$	1177

Yarman Tuning System for turkish music

Degree	Cents	Approximated JI Ratios							Yarman 24-tone tuning System	
1	0	1/1	24	347,122	39/32,11/9,27/22	67	988,708	23/13,16/9	C	0
2	15,092	126/125,100/99,81/80	25	362,215	16/13,100/81,21/17	68	1003,8	16/9,25/14	C#/Db	83
3	30,185	64/63,3125/3072,55/54	26	377,307	31/25,41/33,46/37,5/4	69	1018,893	9/5,59049/32768	C##	142
4	45,277	128/125,36/35,33/32	27	392,399	5/4,64/51,59/47	70	1033,985	29/16,20/11	Dd	191
5	60,369	729/704,28/27,27/26	28	407,491	81/64,19/15,33/26	71	1049,077	11/6	D	204
6	75,461	25/24,117/112,22/21	29	422,584	14/11,23/18,32/25	72	1064,17	37/20,50/27,13/7	D#/Eb	292
7	90,554	20/19,256/243,135/128	30	437,676	9/7	73	1079,262	28/15	D##	347
8	105,646	17/16,16/15,2187/2048	31	452,768	35/27,13/10	74	1094,354	15/8,32/17,17/9	Ed	361
9	120,738	15/14,14/13	32	467,86	38/29,21/16	75	1109,446	256/135,243/128,40/21	E	382
10	135,83	14/13,27/25,13/12	33	482,953	33/25,37/28	76	1124,539	21/11,23/12,48/25	E≠	415
11	150,923	88/81,12/11,35/32	34	498,045	4/3	77	1139,631	27/14,29/15,31/16	F -- E#	498
12	166,015	11/10,54/49	35	513,137	39/29,35/26,27/20	78	1154,723	37/19,39/20,125/64	F#/Gb	579
13	181,107	65536/59049,10/9	36	528,23	19/14,49/36	79	1169,815	49/25,55/28,6144/3125	F##	633
14	196,2	28/25,9/8	37	543,322	26/19,48/35,11/8	80	1184,908	2025/1024,105/53	Gd	695
15	211,292	9/8,26/23	38	558,414	11/8,29/21	81	1200	2/1	G	702
16	226,384	256/225,8/7	39	573,506	25/18,32/23,39/28	14 13 Cent ➔ 128 cent 13 12 Cent ➔ 138 cent 12 11 Cent ➔ 151 cent 8 7 Cent ➔ 231 cent				
17	241,476	144/125	40	588,599	7/5,1024/729,45/32					
18	256,569	37/32,81/70,125/108	41	603,691	24/17,17/12					
19	271,661	7/6	42	618,783	10/7					
20	286,753	33/28,13/11,32/27	43	633,875	23/16,36/25,49/34					
21	301,845	32/27,25/21,81/68	44	648,968	16/11,8192/5625,35/24					
22	316,938	6/5,19683/16384	45	664,06	22/15,69/47,72/49					
23	332,03	63/52,40/33,17/14	46	679,152	37/25,40/27					
			47	694,245	3/2					
			48	701,955	3/2					
			49	717,047	53/35,50/33,1024/675					
			50	732,14	32/21,29/19,75/49					
			51	747,232	192/125,20/13,54/35					
			52	762,324	45/29,59/38,14/9					
			53	777,416	25/16,47/30,11/7					
			54	792,509	30/19,128/81,19/12					
			55	807,601	43/27,8/5,6561/4096					
			56	822,693	37/23					
			57	837,785	34/21,81/50,13/8					
			58	852,878	44/27,18/11,105/64					
			59	867,97	28/17,33/20					
			60	883,062	32768/19683,5/3					
			61	898,155	5/3,42/25,27/16					
			62	913,247	27/16,39/23,17/10					
			63	928,339	128/75,41/24,12/7					
			64	943,431	50/29,216/125,64/37					
			65	958,524	125/72,40/23,47/27					
			66	973,616	7/4,225/128					

Intonation of Maqam Rast in Syria and Egypt							
Notation	Syria 1	Syria 2	Syria 3	Egypt 1	Egypt 2	Egypt 3	Congree
C	0	0	0	0	0	0	0
D	204 $\frac{9}{8}$	204	204	182 $\frac{10}{9}$	204	204	200
E _d	355	344	362	333	350	369	349
F	498 $\frac{4}{3}$	495	498	498	498	498	498
G	702	699 $\frac{3}{2}$	702	702	702	702	702
A _d	853	852	860	837	855	871	851
B _b	996 $\frac{16}{9}$	1003	996	983	996	1018 $\frac{9}{5}$	1010

Refrence: Das arabische Tonsystem von Liberty manik

Intonation of Maqam Rast in Syria and Egypt							
Notation	Syria 1	Syria 2	Syria 3	Egypt 1	Egypt 2	Egypt 3	Congree
C	0	0	0	0	0	0	0
D	204	204	204	182	204	204	200
E _d	355 $\frac{27}{22}$	344 $\frac{39}{32}$	362 $\frac{16}{13}$	333 $\frac{17}{14}$	350 $\frac{27}{22}$	369 $\frac{16}{13}$	349 $\frac{39}{32}$
F	498	495	498	498	498	498	498
G	702	699	702	702	702	702	702
A _d	853	852	860	837	855	871	851
B _b	996	1003	996	983	996	1018	1010

Comparison with 24 equal temperd tuning system

Iranian modern Tar and Setar Tuning System (Hormoz Farhat's research)	Notation	Cent	Tempered (Cent)	Discrepancy (Cent)
	C	0	0	0
	D $\flat$	90	100	10
	D $\natural$	135	150	15
	D	205	200	-5
	E $\flat$	295	300	5
	E $\natural$	340	350	10
	E	410	400	-10
	F	500	500	0
	F $\sharp$	565	550	-15
	G $\natural$	630	650	20
	G	700	700	0
	A $\flat$	790	800	10
	A $\natural$	835	850	15
	A	905	900	-5
	B $\flat$	995	1000	5
	B $\natural$	1040	1050	10
	B	1110	1100	-10
	C	1200	1200	0

Comparison with 24 equal temperd tuning system

Yarman 24-tone tuning System		Tempered (Cent)	Discrepancy (Cent)
C	0	0	0
C#/Db	83	50	-33
C##	142	150	8
Dd	191	200	9
D	204	200	-4
D#/Eb	292	300	8
D##	347	350	3
Ed	361	350	-11
E	382	400	18
E≠	415	400	-15
F -- E#	498	500	2
F#/Gb	579	550	-29
F##	633	650	17
Gd	695	700	5
G	702	700	-2
G#/Ab	788	800	12
G##	852	850	-2
Ad	886	850	-36
A	906	900	-6
A#/Bb	996	950	-46
A##	1042	1050	8
Bd	1070	1050	-20
B	1084	1100	16
B≠	1124	1150	26
c -- B#	1200	1200	0

Conclusion

- Attempt of oriental scientists to find a tuning system, which can describe intonation of that time
- Today's tuning system is deeply rooted in history
- They offered different kinds of tuning systems, valid for different regions, what can be seen also today
- That's why the 24 equal tempered can not represent this diversity specially in case of neutral intervalls.

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Thank you for your attention

